Mr. Hayek and the Classics; A Suggested Interpretation of the Business-Cycle Theory in Prices and Production

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Abstract: This paper endeavors to develop a modern theoretical underpinning of Friedrich August von Hayek’s business-cycle theory as published during the Great Depression in his book Prices and Production. According to Hayek, economic cycles are caused by monetary shocks, which distort the relative-price schedule across economic sectors. Possible consequences of these price distortions, which are also called “Cantillon effects,” include malinvestment and an unsustainable production structure, which sooner or later has to be corrected by a recession. It turns out that this type of economic fluctuation can be condensed into a simple two-sector overlapping generations model.

INTRODUCTION

A collapse in aggregate demand, which is followed by sluggish price adjustments, is probably the most widely cited explanation

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for recurrent boom-and-bust cycles in economic activity. The corresponding business cycle theory was, of course, popularized amid the mass unemployment of the Great Depression through Keynes’s landmark *General Theory*, published in 1936. In a nutshell, Keynes argued that shortfalls between aggregate demand and aggregate supply, which are typically associated with a reluctance to invest and a savings glut, are neither automatically, nor quickly reversed through changes in interest rates, prices, or wages (see, e.g., De Vroey 2016, 3ff.; Niehans 1990, 349ff.). In particular, the price adjustment mechanism can malfunction, because wage reductions or interest rate cuts can lead to deflation, which lures entrepreneurs into postponing investment and, hence, aggravates the downturn. Low levels of interest rates and deflationary policies cannot restore the “animal spirits” of entrepreneurs, to employ Keynes’s famous catchphrase (1936). Rather, to revive aggregate demand by breaking the vicious cycle that depresses investment, a fiscal stimulus is arguably warranted. In contrast to low interest rates through monetary policy, demand activation through fiscal policy is thought to exhibit powerful multiplier effects on investment and consumption and, therefore, has turned into the preferred Keynesian tool for stabilizing macroeconomic activity.

However, according to another contemporary interpretation, the Great Depression was an unavoidable reaction to the overexpansion of the 1920s (De Vroey 2016, 4; Kindleberger 1973, 130). The corresponding theoretical case was probably most prominently made by Friedrich August von Hayek in his book *Prices and Production*, which was published in 1931 and was based on four lectures delivered at the London School of Economics (LSE). In brief, Hayek argued that recessions are necessary evils following any boom which has led to overinvestment and a distorted capital and production structure. More specifically, such distortions in prices and production are thought to be initiated by money and credit expansions. Insofar as newly created money and credit flow via specific sectors into the economy, Hayek suggested that a loose monetary policy is typically associated with a distorted relative-price schedule. Manipulated price signals misguide, in turn, individual consumption and investment decisions and, at least in some sectors, produce an overaccumulation of capital. Such overexpansion leads to an unsustainable production structure. Sooner or later, redundant parts of the capital stock have
to be liquidated, which can arguably only occur through a recession with dampened consumption and divestment. According to this narrative, any form of macroeconomic stabilization policy is futile. In particular, fiscal and monetary stimuli cannot prevent, but only postpone, the inevitable downturn and, possibly, expose the capital and production structure to even greater distortions. In particular, manipulation of monetary variables does no good, insofar as such interventions preserve the mistaken price signals that lie at the origin of boom-and-bust cycles.

Major elements of Keynesian economics, such as the role of inflexible prices and wages, or the temporary lack of market clearing between savings and investment, had already been highlighted by classical economists (see Sowell 1974, chap. 2; Niehans 1990, 54, 59, 103, 349; De Vroey 2011). In a similar vein, the business cycle theory proposed by Hayek drew heavily on earlier contributions to economic theory. Above all, it drew on a detailed account of how money enters the economy via specific sectors, and how corresponding booms could entail relative-price effects on real economic activity, that had already been published by the French economist Richard Cantillon in 1755. In particular, Cantillon observed that new discoveries of monetary metal, such as gold, could initially affect economic activity and prices closely related to the mining sector but are only gradually felt in, e.g., the agricultural sector. This implies that, in relative terms, agricultural prices will temporarily change. These types of relative-price distortions give, in turn, rise to real economic effects (see, e.g., Bordo 1983, 242; Thornton 2006).

Even though Keynes’s and Hayek’s views on economic fluctuations are both rooted in classical economics and partially overlap by, e.g., focusing on movements in savings and investment as the main components of the business cycle, there are also conceptual differences. In particular, Keynes (1936) analysed economic relationships between purely aggregate, or macroeconomic, variables including the overall price and wage level, identified destabilizing downward spirals between prices and economic activity, and advocated fiscal policy as stabilisation tool for an inherently unstable macroeconomic system. Furthermore, in his view, recessions can be avoided when vicious cycles leading to unnecessarily low economic activity are interrupted through adequate economic-policy interventions. Conversely, Hayek (1931) suggested that relative prices
and the composition of consumption, investment, and capital matter more than their aggregate values, highlighted the role of individual savings and investment decisions for economic analysis, suggested that flexible price adjustments act as automatic stabilisers, and interpreted recessions as unavoidable consequences of instable money-and-credit policies, which undermine an inherently stable macroeconomic system.

Keynes (1936) presented a theory without integrating the various economic relationships into a complete model (Patinkin 1990). As the narrative of the General Theory often remains vague, and lends itself to various interpretations, it was followed by a voluminous literature trying to explain what Keynes really meant (see De Vroey 2016, 23ff.). Keynesianism has entered economic textbooks mainly through the IS-LM model of Hicks (1938), whose interpretation was recognized by Keynes (1973, 80) himself (see De Vroey and Hoover 2004). Since this triumphant advance in the late 1930s, this type of the Keynesian theory has led to the New Keynesian model (NKM), which to this day provides probably the most popular framework to analyze short-term interrelationships between economic policy, inflation, and unemployment (see, e.g., Galí, 2015).

Conversely, the type of economic-cycle theory advocated by Cantillon or Hayek has only received sporadic attention, mainly after a credit-boom has ended in a severe recession (see, e.g., Cochran 2010, 2011). From a theoretical point of view, the historical dominance of Keynes (1936) is perhaps surprising, because modern macroeconomic theory has taken up distinct elements of Hayek (1931), such as the insistence on developing macroeconomic theory from individual decision-making, or the recognition that policy interventions can cause, rather than improve, bad economic outcomes (see, e.g., Scheide 1986). However, similar to the original work of Keynes, the largely verbal exposé of Hayek does not always lend itself to a straightforward interpretation. This problem is aggravated by the fact that there have hitherto been virtually no theoretical models to clarify the postulated relationships between relative-price signals, the capital and production structure, and fluctuations in consumption and investment. Possibly the only exception is Beaudry, Galizia, and Portier (2016), who have employed a modern monetary model with search and matching frictions to show that a liquidation of overaccumulated capital can
indeed cause high levels of unemployment, which cannot always be corrected via Keynesian fiscal policy.

Against this background, this article endeavors to contribute to the literature by developing a simple theoretical framework that captures some of the key elements of the cycle theory put forward in Prices and Production. For this task, a model is warranted where individuals as producers and/or consumers decide to save and invest in different forms of capital, where money flows via specific sectors into the economy such that policy shocks can alter the relative-price schedule between these sectors and hence change the consumption, investment, and production structure. Furthermore, the model should be dynamic, such that cyclical adjustments toward its long-term equilibrium can potentially arise. This article suggests that these elements can be found in overlapping generations (OLG) models—one of the main frameworks of modern macroeconomics (see, e.g., Romer 2019, 76ff.)—with two sectors (see Galor 1992). Following Cantillon’s (1755) scenario, the sectors in the model presented herein will be a gold-mining sector that produces monetary metal that provides a store value and an agricultural sector that produces consumption goods (perishable food). Within this context, Cantillon effects will simply originate in extraordinary discoveries of gold, which change the relative prices between the sectors. As will be shown with this two-sector OLG model, relative-price effects can indeed generate cycles in economic activity.

In acknowledgement of the early origin of some elements put forward in Prices and Production, the simple two-sector OLG model shall be referred to as the Cantillon-Hayek cycle (CHC) theory, but this label should not disguise its obvious overlap with the Austrian business cycle (ABC) theory, as discussed by, e.g., Cochran (2010, 2011) in light of the global financial crisis (see also Hébert 1985). A key difference, however, is that the ABC theory typically emphasizes the destabilizing effects of monetary policy and credit creation in a fractional reserve banking system (see, e.g., Hébert, 1985, 275ff.; Cochran, 2011, 271–72). In contradistinction, in the model developed in this study the role of the money and banking sector is ignored.

Furthermore, Prychitko (2010) and Mulligan (2013) suggest that the ABC theory overlaps, in turn, with Minsky’s financial instability hypothesis.
This article is organized as follows: The first section reviews the CHC theory and provides an overview of the relevant literature. Section 2 develops the simple model reflecting the principal elements of this theory. The final section provides some concluding remarks.

I. THE CANTILLON-HAYEK CYCLE THEORY IN WORDS

As Hayek (1931, chap. 1) himself emphasized, he did not develop his economic cycle theory from scratch, but drew heavily on earlier economic thought. Among other contributions, he refers to the quantity equation in David Hume’s 1752 Political Discourses, the relative-price effects in Cantillon’s 1755 Essai Sur La Nature Du Commerce En Général, the impact of the quantity of money upon interest rates and prices as discussed in Henry Thornton’s 1802 “Paper Credit of Great Britain,” and the role of the natural rate of interest for economic stability in Knut Wicksell’s 1898 Geldzins und Güterpreise (see also Niehans 1990, 24ff., 53ff., 105ff., 247ff.). Furthermore, reflecting Hayek’s personal and intellectual origin in Vienna, stepping-stones for his cycle theory were laid by fellow Austrian economists, especially Ludwig von Mises with his 1912 in-depth verbal discussion of the functions, forms, and the value of money, including its interrelationships with credit and relative prices. In particular, Mises’s (1912, part 2, chap. 6) analysis of the role of relative-price effects as regards current “consumption goods” and “investment goods,” e.g., those that are not destined for current consumption, is singled out by Hayek (1931, 25–26) as an important ingredient in his cycle theory. However, many of these ideas were only introduced to an English-speaking audience through Hayek’s 1931 Prices and Production. This book makes a contribution in its own right by integrating the abovementioned strands of the literature to argue that relative-price effects can

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2 The terminology for goods that are destined for current consumption and future consumption is not uniform between Mises and Hayek. Mises (1912, part 2, chap. 6, section 1) refers to “present goods” (“gegenwärtige Güter”) and “future goods” (“künftige Güter”), while Hayek (1931, 25, 36–37) refers to “consumers’ goods” and “producers’ goods.”

3 Elements of Prices and Production first appeared in German in Hayek (1928a, 1928b, 1929a, 1929b).
alter the production structure such that money and credit booms generate economic fluctuations (see, e.g., Ekelund and Hébert 1997, 515–16). During the 1930s, partially as a response to points of criticism raised by Keynes and his disciples, Hayek elaborated on his cycle theory (see, e.g., Wapshott 2012). Landmark contributions toward this debate include “Monetary Theory and the Trade Cycle” (1933), “Profits, Interest, and Investment” (1939), and “The Pure Theory of Capital” (1941). Finally, when high inflation had turned into a major problem, Hayek (1979) revisited his cycle theory, but focused on the role of price stability (see White 1999; Cochran 2011).

The quantity theory serves as the point of departure for the theoretical analysis in *Prices and Production*. It is indeed uncontroversial that, in a fully monetized economy and over any given period, the aggregate value of payments is by definition equivalent to the aggregate value of production, which implies an intimate relationship between the money stock, the overall velocity of money, the general price level, and total production. However, whereas Keynes (1936, chap. 20, section 3) found the quantity equation wanting because it can break down during periods with deficient aggregate demand, Hayek (1931, 5ff.) argued that relationships between aggregate money, overall inflation, and total production disguise the crucial role of disaggregate prices and the structure of production in a multisector economy. Heterogenous developments at the individual level are, arguably, crucial for understanding the disturbing effects of economic cycles. The distinction between an aggregate and a disaggregate theory cuts into fundamental methodological issues as regards the appropriate level of economic analysis and the role of individuals as decision-makers. For example, Hayek (1931, 4–5) lambasted a naïve interpretation of the quantity theory as an attempt “to establish *direct* causal connections between the *total* quantity of money, the *general level* of all prices and, perhaps, also the *total* amount of production.”

He goes on to suggest that this is inadequate because none of these magnitudes *as such* ever exert an influence on the decision of individuals; yet it is on the assumption of a knowledge of the decision of individuals that the main propositions of ... economic theory are based.... In fact, neither aggregates nor averages do act upon one another, and it will never be possible to establish necessary connections of cause and effect between them as we can between individual
phenomena, individual prices, etc. I would even go so far as to assert that, from the very nature of economic theory, averages can never form a link in its reasoning. (Hayek, 1931, 4–5)

This paragraph reflects the key tenets of Austrian economics that decisions are subjective and are made by individuals who differ in motives, knowledge, or expectations (see, e.g., Ekelund and Hébert 1997, 508ff.).

Launching an economic analysis from the individual level can have far-reaching implications. Above all, under a disaggregate view, shocks to, e.g., money and credit do not directly affect overall inflation, but impact first and foremost specific prices (including certain wages and interest rates). Furthermore, unless the economy involves completely homogenous individuals, these shocks are typically transmitted to prices and production in a heterogeneous manner. In particular, regardless of whether we contemplate an increase in the amount of currency through monetary policy interventions or privately created deposits by commercial banks, the added money and credit flows via specific sectors into the economy and is typically spent by select individuals on certain classes of goods, services, and assets. Taken together, individual heterogeneity in a disaggregated economy implies that monetary shocks can give rise to so-called relative-price effects. The view that across a range of products nominal prices will change at uneven rates, and that the associated relative changes entail real economic effects, can be traced back to Cantillon (1755, part 2, chap. 6). In particular, Cantillon described how new discoveries of monetary metal within a purely metallic currency system initially benefit

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4 In contrast, in the preface to the French edition of *The General Theory*, Keynes (1942) seems to argue that there is no major difference between modeling individual decisions and relationships between macroeconomic aggregates:

I regard the price level as a whole as being determined in precisely the same way as individual prices; that is to say, under the influence of supply and demand. Technical conditions, the level of wages, the extent of unused capacity of plant and labour, and the state of markets and competition determine the supply conditions of individual products and of products as a whole. The decisions of entrepreneurs, which provide the incomes of individual producers and the decisions of those individuals as to the disposition of such incomes determine the demand conditions. And prices—both individual prices and the price-level—emerge as the resultant of these two factors.
the gold miners, whereas the new bullion and coins trickle only gradually through to other sectors, such as agriculture, and hence alter relative food prices in the process (see also Niehans 1990, 31–33).\footnote{Cantillon effects can be invoked against the view that the quantity theory necessarily implies the neutrality of money when prices are flexible. In particular, Cantillon (1755, part 2, chap. 7) argued that money is not per se neutral with respect to (flexible) prices, because money does not affect equally all the kinds of products and merchandise, proportionally to the quantity of money, unless what is added continues in the same circulation as the money before, that is to say unless those who offer in the market one ounce of silver be the same and only ones who now offer two ounces when the amount of money in circulation is doubled in quantity. (qtd. in Thornton 2006, 48)} Bearing witness to their historical origin, the relative-price effects from monetary shocks are also called “Cantillon effects” (see, e.g., Bordo 1983, 242; Thornton 2006, 47ff.).

Cantillon effects are obviously not restricted to a society of miners and farmers. For example, Malthus (1811) said of an increasing circulation of paper money (or notes) that relative-price effects can arise between individuals who currently produce and consume and individuals who only consume. In his words:

If a thousand millions of notes were added to the circulation, and distributed to the various classes of society exactly in the same proportions as before, neither the capital of the country, nor the facility of borrowing, would be in the slightest degree increased. But, on every fresh issue of notes, … a larger proportion falls into the hands of those who consume and produce, and a smaller proportion into the hands of those who only consume. And as we have always considered capital as that portion of the national accumulations and annual produce, which is at the command of those who mean to employ it with a view to reproduction, we are bound to acknowledge that an increased issue of notes tends to increase the national capital. (Malthus 1811, 364–65)

Why would relative-price effects matter for aggregate economic fluctuations? In this regard, Hayek (1931, chap. 2, chap. 3) observes that prices not only fulfill a compensation function in individual transactions, but also act as an information and coordination device by indicating economic scarcity and sending signals organizing economic activity. Hence, manipulated prices can misguide
individual decisions and, in turn, distort the capital and production structure of the economy. Above all, misleading money and credit policies have an immediate effect on interest rates and investment decisions. These manipulations are not innocuous: they lead to an unsustainable production structure, which makes an economy more and more prone to a crisis. In particular, an indiscriminate creation of money and credit tends to push interest rates below their equilibrium level—or what Wicksell (1898) called the natural rate. Low levels of interest rates can foster, in turn, investment in relatively capital-intensive sectors (Hayek 1931, 86–87; 37ff.). Borrowing heavily from Austrian capital theory—and employing the corresponding terminology—Hayek (1931) devotes chapter 2 to describing how money and credit booms can guide economic activity toward a “longer,” “more roundabout,” or “more capitalistic” production structure. In modern terminology, this probably refers to investments in goods whose returns come in the relatively distant future (see Steele, 1992, 478ff.). When contemplating present value calculations, it is indeed conceivable that, e.g., low interest rates increase the range of profitable investment projects (see Steele 1992, 479).

Typically, a shift toward a more capitalistic production structure—in terms of an increasing output of “investment goods”—comes at the expense of sectors whose output consists of current “consumption goods” (Hayek 1931, 88). Insofar as the money and credit boom is an exogenous event, individuals are essentially forced to live with a lower amount of current consumption goods to “set aside” the savings that are needed to support the investment boom. It is again noteworthy that this doctrine of “forced savings” can be traced back to classical writings, e.g., Malthus (1811, 364) and Thornton (1802, 263) (see also Hayek 1932; Sowell 1974, 65). However, Hayek connected the forced savings doctrine with the abovementioned distinction between individuals who produce and consume (or entrepreneurs), and individuals who only consume. In particular, as regards the reduction in the available amount of consumption goods when moving toward a more capitalistic production structure, he observed that

this sacrifice is not voluntary…. It is made by the consumers in general who, because of the increased competition from the entrepreneurs who have received the additional money, are forced to forgo part of what they used to consume. It comes about not because they want to consume
less, but because they get less goods for their money income. There can be no doubt that, if their money receipts should rise again, they would immediately attempt to expand consumption to the usual proportion. (Hayek 1931, 57)

In other words, relative-price effects can generate a production structure with overinvestment and underconsumption. However, when the money and credit expansion slows down, or is even reversed, the misallocation between investment and consumption goods will be corrected (Hayek, 1931, 89ff.). Arguably, this correction is necessarily associated with an economic downturn (Hayek 1931, 92–93; Hayek 1979, 25). Taken together, a distorted production structure is unsustainable, as

the machinery of capitalistic production will function smoothly only so long as we are satisfied to consume no more than that part of our total wealth which under the existing organisation of production is destined for current consumption. Every increase in consumption, if it is not to disturb production, requires previous new saving…. If the increase of production is to be maintained continuously, it is necessary that the amount of intermediate products in all stages is proportionally increased…. The impression that the already existing capital structure would enable us to increase production almost indefinitely is a deception. (Hayek 1931, 95)

The policy conclusions of the CHC theory are diametrically opposed to the Keynesian belief in the merits of government intervention to stabilize the economy. According to Hayek, policies such as monetary expansions and fiscal stimuli are not the solution but rather the cause of economic instability. To recapitulate, manipulated price and interest rate signals interfere with individual investment and consumption plans. Misguided individual consumption and investment decisions bestow an economy with a distorted production and capital structure. Insofar as a money-and-credit boom is typically associated with an overexpansion, which has eventually to be corrected by a liquidation of capital, fiscal or monetary stimuli cannot prevent a downturn from happening (Hayek 1931, 97ff.). Rather, such government interventions are problematic, because they preserve, or even aggravate, the distorted price signals and, thereby, tend to prolong and/or deepen the recession (see also Beaudry, Galizia, and Portier 2018, 119–20).
Economic downturns are necessary evils, and recoveries require a restoration of interest rate and price signals, based on which investments in a sustainable production and capital structure can made (Hayek 1931, 99).

According to the CHC theory, the only way to dampen economic fluctuations is to stabilize money and credit conditions (Hayek 1931, 97ff; Hayek 1939, 73–82; Hayek 1979, 4). In this way, Cantillon effects and distorted production structures do not occur in the first place and unnecessary large swings in investment and savings are avoided. However, it is not entirely clear what stable monetary conditions concretely mean. Hayek (1931, chap. 4) refers to upholding the convertibility of the currency at the established mint pars of the gold standard, but after the transition to a pure fiat currency during the 1970s resulted in high inflation, Hayek (1979) turned to price stability as the key criterion (see White 1999; Cochran 2011).

II. MORE THAN WORDS: A SIMPLE TWO-SECTOR MODEL OF THE CANTILLON-HAYEK CYCLE THEORY

2.1. Background

For a modern economist who has read the purely verbal exposés of Cantillon (1755) or Hayek (1931), it is probably not always clear how exactly relative-price effects can alter the capital and production structure such that boom-and-bust cycles arise. What determines the long-term equilibrium with respect to which concepts such as “overinvestment” are defined? Can an economic boom indeed be followed by cyclical adjustments toward that equilibrium and, if so, what assumptions are required to obtain this result? These and other questions can only be answered by means of a theoretical model.

To capture the key ideas of the CHC theory, a microfounded model is warranted that lends itself to introducing a money-like asset, encompasses several forms of capital, includes separate

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6 Hence, like monetarism, the CHC theory interprets cycles as monetary phenomena. However, the monetary distortions occurring at the disaggregate level in Hayek (1931) stand in sharp contrast to the overarching role attributed to monetary aggregates in, e.g., Friedman and Schwartz (1963).
sectors producing investment and consumption goods, allows for relative-price changes that give rise to Cantillon effects, and distinguishes between individuals who primarily produce and individuals who primarily consume. Furthermore, the different sectors and individuals should be more or less directly affected by monetary expansions, and the model should be dynamic in order to determine whether the adjustment toward some long-term equilibrium occurs in a cyclical manner. Arguably, these elements can be found in two-sector overlapping generations (OLG) models pioneered by Galor (1992) and discussed in Azariadis (1993, 258–67), Farmer (1997), Farmer and Wendner (2003), and Cremers (2006). In particular, a standard (one-sector) OLG model lends itself to the introduction of a medium of exchange à la Samuelson (1958), accounts for the allocation between consumption and investment, encompasses different groups of individuals (“generations”), and embodies the concept of the steady state as long-term equilibrium. Furthermore, when an OLG model encompasses two sectors, the relative price of investment and consumption goods associated with these sectors can potentially change.

What is particularly relevant in the context of this study is Farmer and Wendner’s (2003) suggestion that two-sector OLG models can exhibit cyclical adjustment patterns after a policy shock. However, Farmer and Wendner (2003), as well as Galor (1992), focus on the role of economic growth and Cremers (2006) on the role of dynamic inefficiency in a two-sector economy. Consequently, these papers neglect issues related to business cycles, which Hayek (1931) emphasized.

Against this background, this section endeavors to develop a simple model to show how relative-price effects can, under certain parameter sets, give rise to economic cycles in a two-sector OLG environment. To keep the model simple and tractable, capital will be the only production factor (there is no labor market), and the effects of time discounting, population growth, and technological progress are ignored. Finally, specific production functions are imposed.\(^7\) In

\(^7\) Thanks to these simplifications, it is possible to avoid such issues as multiple equilibria, which can arise in an OLG environment and have been used to study business cycles (see, e.g., Grandmont 1985). Cycles associated with multiple equilibria are typically not attributed to shocks or variations in economic policy and, hence, do not reflect the CHC theory.
particular, the two-sector OLG model with a Cobb-Douglas-Leontief technology (Farmer 1997; Farmer and Wendner 2003) will be extended to a constant elasticity of substitution (CES)–Leontief economy. In the current context, the flexibility of the CES function is needed in order to compare the different reactions of capital inputs to relative-price changes across a range of production technologies. Of course, Cantillon’s agricultural and gold-mining sectors hardly account for the roles of monetary policy in the manipulation of interest rates or of the commercial banking sector in creating unstable credit booms, as emphasized by the ABC theory. Also, the CES-Leontief economy only hints at the lengthening of the production structure, as discussed by Hayek (1931, chap. 2). Nevertheless, the two-sector OLG model reflects a standard framework in modern macroeconomics, and can apparently capture the link between relative-price manipulations between different economic sectors, changes in the capital structure, and cyclical adjustments toward a new equilibrium.

2.2. Notation and Basic Assumptions

The present OLG model encompasses two forms of capital. Variables, e.g., physical and land capital, pertaining to these forms, are represented by superscripts $i$ and $j$. There are two economic sectors. Variables pertaining to these sectors are denoted by superscripts $a$ and $g$. Subscript $t$ refers to time periods.

The $a$ sector is like agriculture in Cantillon’s (1755, part 2, chap. 6) example. In particular, in each period $t$, this sector employs both forms of capital, e.g., $k^a_t$ and $k^j_t$, to produce a nondurable consumption good, $y^a_t$.

The $g$ sector employs both forms of capital, e.g., $k^{gi}_t$ and $k^{gj}_t$, to produce a pure investment good, $x^g_t$, which cannot be consumed. In concrete terms, the $g$ sector is like gold mining in Cantillon’s (1755, part 2, chap. 6) example.

Although the two forms of capital are not sector specific, they differ insofar as some forms of capital are endowed and others can be produced. In particular, there is a fixed endowment of $j$-form capital that does not depreciate (e.g., constant land capital). To simplify the model, this endowment is assumed to be $k^j=2$. It is
also assumed that $j$-form capital is perfectly mobile and is allocated between the sectors according to

\[(1) \quad k^j = k_t^{aj} + k_t^{aj} = 2\]

Conversely, it is assumed that $i$-form capital is perfectly immobile between the sectors. To simplify the analysis, the endowment of $i$-form capital in the $a$-sector is normalized to one, that is, $k_t^{ai} = 1 \forall t$. However, $i$-form capital in the $g$-sector is assumed to depreciate fully at the end of period $t$, but can be augmented through the production of investment goods ($x_t^g$). Hence, the corresponding capital accumulation function equals

\[(2) \quad k_t^{gi} = x_t^g\]

Prices pertaining to goods produced in the $a$ sector and the $g$ sector are denoted by, respectively, $p_t^a$ and $p_t^g$.

**Relative price:** the relative price between a sector (consumption) and $g$ sector (investment) goods is defined as

\[(3) \quad p_t = \frac{p_t^g}{p_t^a}\]

With relative prices, such as $p_t$, one price can be chosen as numéraire. It is here assumed that $\bar{p}_t^g = 1$.

Note that the relative price $p_t$ will be required to express values in the same unit. Where necessary, prices will be converted into $a$ sector units.

**Remark 1 (relative-price effects):** fluctuations of relative prices (modeled by equation [3]) are at the heart of the CHC theory, as they capture the Cantillon effects that are supposed to induce boom-and-bust cycles (see section 1). In particular, such relative-price effects can originate in a shock to, or manipulation of, the current $g$ sector price, i.e., the numéraire. For example, an increase of $\bar{p}_t^g$, which implies an increase in $p_t$, signals that goods in the $g$ sector (i.e., gold) have become relatively more expensive.

A representative individual enters the economy at time $t=0,1,2,\ldots$ and exits at $t+1$. As there is no population growth, variables coincide with their per capita values. However, during period $t$, individuals own the fixed stock of $j$-form capital and are pure producers of investment goods $x_t^g$ and consumption goods $y_t^g$. During period $t+1$, individuals are pure consumers of an amount denoted by $c_{t+1}$. 
Remark 2 (heterogenous population): the overlapping structure just mentioned implies that during each period \( t \), the population consists of a group of (pure) producers, and a group of (pure) consumers.

2.3. Assumptions about the Production Functions

The production of consumption goods is assumed to obey a simple Leontief function with both forms of capital as factor inputs. With \( k_t^{a} = 1 \) (see section 2.2), that function is

\[
y_t^a = \min(1, k_t^{aj})
\]

The rigid production structure of Leontief functions simplifies the analysis by limiting the output of consumption goods in the \( a \) sector to one unit. Furthermore, Leontief technologies typically require a fixed combination of factor inputs (here only capital) to optimally produce a given amount of output. Specifically, to produce the maximal amount of consumption goods with function (4), the optimal capital input in the \( a \) sector would be fixed to 1, that is,

\[
k_t^{aj} = 1
\]

It is assumed that investment goods in the \( g \) sector are produced by means of a constant elasticity of substitution (CES) function given by

\[
x_t^g = \left( (k_t^{gi})^\rho + (k_t^{gj})^\rho \right)^{-\frac{1}{\rho}}, \text{with } 0 < \nu \leq 1 \text{ and } -\infty < \rho \leq 1
\]

Within the current context, this production function is useful, because it encompasses a range of technologies to produce investment goods, which are typically the main channel through which fluctuations occur in the CHC theory (see section 1). Specifically, \( \nu \) reflects whether or not the production of \( x_t^g \) is subject to scale economies, where \( \nu = 1 \) yields constant returns and \( 0 < \nu < 1 \) decreasing returns to scale.\(^8\) Furthermore, \( \rho \) is a substitution parameter, which determines the CES, denoted by \( \sigma \), between the inputs of different forms of capital via \( \sigma = 1/(1-\rho) \). When \( 0 < \rho < 1 \), there is a high elasticity of substitution (e.g., \( \sigma > 1 \)). When \( \rho < 0 \), the CES is \( \sigma < 1 \), which implies that the capital structure that produces

\(^8\) Increasing returns to scale would arise in (6), if \( 1 < \nu \). However, because capital is here the only production factor, this case seems implausible.
$x_t^g$ is rather rigid. Special cases arise when $\rho$ approaches 1 (and $\sigma=\infty$), which yields a linear; when $\rho$ approaches 0 (and $\sigma=1$), which yields a Cobb-Douglas; and when $\rho=-\infty$ (and $\sigma=0$), which yields a Leontief production function.$^9$

Under a high degree of substitutability between the different forms of capital, as measured by $\rho$, it will be more likely that relative-price effects will give rise to a distorted production structure and, in turn, economic cycles. Conversely, with a Leontief technology, e.g., $\rho=-\infty$, the two forms of capital are perfect complements and typically enter (6) in fixed proportions. In this scenario, relative-price changes do not affect the capital structure in the $g$ sector at all and are hence unlikely to initiate economic cycles.

2.4. The Saving and Consumption Decisions

In the current two-sector OLG model, the saving decision is trivial.

Remark 3 (forced savings): Any individual is initially a pure producer and becomes a pure consumer during the next period (see section 2.2). This assumption reflects the concept of “forced savings,” as individuals have no other option but to save their income to satisfy future consumption (which shall enter into the standard utility function, $u(c_t)$). They cannot shift consumption across time or postpone productive activity.

Consumption is subject to the budget constraint. Specifically, as a pure producer during period $t$, an individual generates income from producing investment goods, $x_t^g$, and consumption goods, $y_t^g$. Savings, denoted by $s_t$, are given by the difference between the current output and expenditures for buying $i$-form capital in the $g$ sector at price $p$, from current pure consumers. Hence, the budget constraint of the pure producer during period $t$ equals

$$(7) \quad \frac{x_t^g}{p_t} + y_t^a - \frac{k_t^g}{p_t} = s_t$$

where $p_t$ harmonizes price units.

At the aggregate level, which encompasses the producer and consumer during period $t$, savings are determined by the difference

$^9$ See, e.g., Varian (1992, 13–20).
between output (of investment and consumption goods) and consumption, that is,

\[ x_t^a + y_t^a - c_t = s_t \]  

(8)

Because consumption goods are nondurable (e.g., perishable food), they cannot be stored. Hence, in each period, the market-clearing condition equates consumption \( c_t \) with the output of consumption goods:

\[ c_t = y_t^a \]  

(9)

Inserting (9) into (8) yields

\[ \frac{x_t^a}{p_t} = s_t \]  

(10)

which reflects the usual aggregate equivalence between investment, which is valued at the relative price, and savings. Inserting (10) back into (7) yields

\[ y_t^a = \frac{k_t^a}{p_t} \]  

(11)

An interpretation of (11) is that \( i \)-form capital in the \( g \) sector, which is produced from past investment goods \( x_{t-1}^g \) according to (2), encapsulates the option to buy current consumption goods at relative price \( p_t \). The values \( k_t^a/p_t \) and \( y_t^a \) concurring with such a transaction are necessarily determined through bargaining between the consumer and the producer. To pin down these values, assume that the pure consumer can make the pure producer a take-it-or-leave-it offer. It is well known that under this bargaining arrangement, the pure consumer can extract all the gains from trade (see, e.g., Nosal and Rocheteau 2011, 61ff.). In the current model, this implies that the consumer will demand the maximum output of \( y_t^a \) to maximize his utility, \( u(c_t) \), with \( y_t^a = c_t \) (see (9)). Because there is a one-unit endowment of \( i \)-form capital in the \( a \) sector, the maximum output of consumption goods in (4) equals \( y_t^a = 1 \). Furthermore, according to (5), a one-to-one capital input is required to optimally produce \( y_t^a = 1 \). Taken together, we have:

\[ y_t^a = k_t^ai = k_t^aj = 1 \forall t \]  

(12)

For the sake of simplicity, it is henceforth assumed that the conditions hold that stabilize the output of consumption goods as well as the corresponding capital inputs at one unit. This concurs with the CHC theory insofar as cycles in economic activity are primarily attributed to movements in the investment goods sector.
2.5. Capital Allocation and Production Structures of Different Lengths

Because $i$-form capital is immobile, its allocation is not guided by an intersectoral arbitrage condition. Conversely, producers can freely allocate $j$-form capital between the sectors. On capital markets with perfect intersectoral mobility (see section 2.2), arbitrage transactions equalize the marginal effect of $j$-form capital upon the revenue to produce investment goods in the $g$ sector, denoted by $R_t^g$, and consumption goods in the $a$ sector, denoted by $R_t^a$; that is,

\[ \frac{\partial R_t^g}{\partial k_t^{aj}} = \frac{\partial R_t^a}{\partial k_t^{aj}} \tag{13} \]

Recall from section 2.2 that $j$-form capital is owned by the pure producers and, thus, not subject to a rental price. Therefore, the revenue in the $a$ sector is simply given by $R_t^a = y_t^a$. With the Leontief technology of (4), the output of consumption goods equals

\[ y_t^a = \begin{cases} k_t^{aj} & \text{if } k_t^{aj} < 1 \\ 1 & \text{otherwise} \end{cases} \tag{14} \]

The properties of (14)—especially its marginal product of capital—depend on how the actual combination of capital compares with its optimal input. As long as $k_t^{aj}$ (e.g., agricultural land) is the limiting production factor, (14) implies that

\[ \frac{\partial R_t^a}{\partial k_t^{aj}} = 1 \tag{15} \]

The $g$ sector revenue is given by $R_t^g = x_t^g / p$, where $p$ is needed to harmonize price units. By substituting the production function (6) for $R_t^g$ and employing (13) and (15), a consolidated production function for investment goods that only depends on $p_t$ is derived (see appendix A) and is given as

\[ x_t^g = \left( \frac{p_t}{v} \right)^{\frac{v}{v-\rho}} \tag{16} \]

According to (16), when $v-\rho > 0$, a higher value of $p_t$ (which implies that the relative $g$ sector price has increased) leads to a larger output of $x_t^g$. When the returns to scale effect of $v$ in the production function (6) exceeds the substitution effect of $\rho$, an increase in the relative $g$ sector price expands the output of investment goods. Conversely, when $v-\rho$ in the denominator of the exponent of (16) is negative, the substitution away from
produced capital in the g sector dominates, and an increase in $p_t$ reduces the output of $x_t^g$.

In any case, as $x_t^g$ determines the capital stock ($k_{t+1}^g$) according to (2), changes in $p_t$ affect period $t+1$, the period when the current producer has become a pure consumer. Furthermore, capital is a variable in the g sector production function (6) of the future producer. Hence, although the g sector does not produce a consumable good, the investment good ($x_t^g$) can be used as a potential medium of exchange for future claims on consumption goods ($c_{t+1}$). Taken together, in the spirit of Samuelson (1958), as long as individuals expect a positive future g sector price, the corresponding output can be valuable, even when investment goods never enter the utility function (see also Sargent and Ljungqvist 2012, 326ff.). However, rather than contemplating a given endowment of fiat money, in this model the medium of exchange has to be reproduced during each period.

**Remark 4 (different production structures):** the production structures of the a and g sectors differ. In particular, using the terminology of Hayek (1931, 32ff.), the g sector has a “long” structure in the sense of producing investment goods, which provide a way to satisfy future consumption. Conversely, the production structure of the a sector is “short” in the sense of employing current capital to produce current (nondurable) consumption goods.10

### 2.6. Capital and Relative-Price Dynamics and the Steady State

Because the output of consumption goods in the a sector is fixed by (12), the dynamics of the current two-sector model are governed by the production of investment goods, which depends primarily on the evolution of i-form capital in the g sector. Let the initial value be given by $k_0^g$ and the initial relative price by $p_0$. Jointly, the capital accumulation function of (2); the link between relative prices, consumption, and capital of (11); the stable output of consumption goods of (12); and the consolidated production function of (16) yield

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10 Because $k_t^g$ depreciates completely at the end of each period $t$, the current model cannot fully account for the concept of a ”lengthening of the production process”. Furthermore, as investment goods $x_t^g$ merely provide a medium to transfer value to the next period, they cannot generate an increase of productivity by ”roundabout methods of production”.
\( (17) \quad k_{t+1}^{g_i} = \left( \frac{k_t^{g_i}}{v} \right)^{\frac{v}{v-\rho}} \)

Taken together, the interaction between capital, \( k_t^{g_i} \), and relative prices, \( p_r \), through (11) and (16) lies at the heart of the dynamics of the current two-sector OLG model. Indeed, below it will be shown that, depending on the parameter set, (17) can give rise to cyclical dynamics. However, before turning to the dynamic properties of (17), its long-term equilibrium is defined in terms of the steady state values for \( k_t^{g_i} \) and \( p_r \) (\( g_t^{g_i} \) and \( \bar{p} \)) in proposition 1.

**Proposition 1 (steady state):** equation (17) exhibits a nontrivial steady state of \( 0 < \bar{k}^{g_i} \), given as

\( (18) \quad \bar{k}^{g_i} = (\nu)_{\bar{p}} \)

The corresponding steady state value for \( p_r \) is given as

\( (19) \quad \bar{p} = (\nu)_{\bar{p}} \)

(See appendix B for proofs.)

The steady states \( 0 < \bar{k}^{g_i} \) and \( 0 < \bar{p} \) occur when \( 0 < \nu \).

### 2.7. Converging Cycles

Can the current two-sector OLG model generate boom-and-bust cycles as postulated by the CHC theory? The answer depends on the dynamic properties of (17), which determine the development in the \( g \) sector. In particular, the dynamic behavior of relative prices (\( \bar{p} \)) follow from (11) and (12), and that of the production of investment goods in the \( g \) sector from (16).

To solve the nonlinear first-order dynamic equation of (17), the first-order Taylor approximation is derived at the steady-state value \( \bar{k}^{g_i} \) of (18) (see appendix C), which yields

\( (20) \quad \left[ k_{t+1}^{g_i} - \bar{k}^{g_i} \right] = \frac{\nu}{v-\rho} \left[ k_t^{g_i} - \bar{k}^{g_i} \right] \)

Depending on whether the term \( \nu/(v-\rho) \) of (20) is positive or negative, and whether or not this term has an absolute value that is greater or smaller than 1, the adjustment path of \( k_t^{g_i} \) can be smooth or cyclical as well as convergent or explosive (see, e.g., Azariadis 1993, 33ff.; Chiang 1984, 505ff.). Typically, a set of parameters with
-1 < \frac{\nu}{(\nu - \rho)} < 0 \text{ is warranted to obtain the convergent cycles postulated by the CHC theory. Proposition 2 clarifies when this scenario arises.}

**Proposition 2 (stable cyclical adjustments):** in the two-sector OLG model underpinning the dynamic equation (17), \( i \)-form capital \( (k^g_t) \) moves in cycles toward the steady state of \( \bar{k}^g \) when

\[ 0 \leq \nu < \rho \leq 1. \]

When \( \nu < \frac{\rho}{2} \), the corresponding cycles are convergent (e.g., nonexplosive; see appendix C for proofs).

Hence, stable cycles arise only under certain parameter sets. Above all, the substitution parameter \( (\rho) \) and economies of scale \( (\nu) \) of production function (6) for investment goods matter. Figure 1 depicts the different dynamic behavior across the permissible parameter values of \( -\infty < \rho \leq 1 \) and \( 0 < \nu \leq 1 \). In particular, the gray area highlights combinations of \( \rho \) and \( \nu \) giving rise to cyclical dynamics and the hatched area combinations resulting in convergent (nonexplosive) dynamics.

**Figure 1. Dynamic Properties of (17) with Different Values of \( \rho \) and \( \nu \)**

![Diagram showing dynamic properties with different values of \( \rho \) and \( \nu \)]

Proposition 2 and figure 1 have established that \( k^g_t \) follows a cyclical adjustment path when the substitution parameter is positive and larger than the returns to scale parameter of the g sector.
production function (6). This result is, perhaps, intuitive, because the high substitutability between the two forms of capital for the production of investment goods \((x_t^g)\) implies that these structures can react markedly to relative-price changes (i.e., the Cantillon effects are quite strong). Furthermore, when the substitution effect is larger than the returns to scale effect, according to discussion around (16), an increase in \(p_t\) reduces the output of \(x_t^g\) and, in turn, \(k_t^{gl}\). This provides the basis for a cyclical interaction between prices and capital output. Conversely, noncyclical adjustments necessarily arise when \(\rho \leq 0\), e.g., when capital inputs are rather complementary.

Figure 2 illustrates the main result by showing numerical examples for a (stable) cyclical and a noncyclical adjustment of \(p_t\), \(x_t^g\), and \(k_t^{gl}\) to a shock to relative prices in period \(t = 1\). In particular, a positive shock to \(p_t\) is considered, meaning that the relative \(g\) sector price increases (see remark 1). When the different forms of capital are highly substitutable, as in example 1 with \(\rho = 0.8\), this relative-price shock decreases the current output of investment goods \((x_t^g)\) according to (16) and, subsequently, \(k_t^{gl}\) according to (2). As a reaction to this development, future relative prices decline and subsequent cycles between capital and relative prices arise. Conversely, when lowering the substitution parameter to \(\rho = -0.8\) in example 2, there are no cycles, because the capital structure in the \(g\) sector is rather rigid, and the initial increase in \(p_t\) is followed by a smooth regression to the original level.

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11 This type of price-quantity interaction has been widely documented for the cobweb model (also known as the “hog cycle”). For a textbook discussion of the cobweb model, see Chiang (1984, 561–65).

12 When the stability condition \(\nu < \rho/2\) is violated, the interaction between \(p_t\) and \(k_t^{gl}\) produces nonconvergent cycles.

13 Again, a noncyclical adjustment can occur in a convergent or nonconvergent manner (see figure 1).
SUMMARY AND CONCLUSION

This paper suggests that the cycle theory described verbally by Friedrich August von Hayek—and in a rudimentary form much earlier by Richard Cantillon—can be expressed through a simple overlapping generations model. In particular, when two sectors are introduced into the OLG model, it is possible for economic shocks to alter the relative prices of goods associated with these sectors. This can lead to a reorganization of the production structure and subsequent boom-and-bust cycles. Hopefully, presenting the Cantillon-Hayek theory using a modern macroeconomic model clarifies the underlying narrative for audiences that are perhaps unfamiliar with the original verbal discussions and helps uncover
the different answers to seminal questions in business cycle research when compared with the Keynesian theory.

The Cantillon-Hayek cycle theory offers vastly different answers to enduring questions about the nature of business cycles, such as the disturbances that cause fluctuations in economic activity. According to the Keynesian view, demand shocks are paramount. Conversely, in the Cantillon-Hayek theory, economic fluctuations originate in excessive monetary expansion that distorts the price schedule and misdirects investment toward capital-intensive sectors. This leads to an overaccumulation of certain forms of capital, which must eventually be undone through a recession. Furthermore, economic expansions and recessions typically persist for some period of time. Hence, the question of what causes this persistence arises. Whereas Keynesians emphasize the role of price stickiness, in the Cantillon-Hayek theory, cycles are not immediately eliminated due to the delays in reorganizing the capital stock, which implies that booms and busts can become entrenched. Finally, why can nominal variables, such as money, have real effects? To explain this, Keynesians invoke sticky prices. By contrast, even when individual prices are fully flexible, the Cantillon-Hayek theory recognizes that money can flow via specific sectors into the economy. Hence, prices of goods closely associated with the economic sectors through which nominal expansions occur can change relative to other prices. Temporarily, such “Cantillon effects” can have real economic consequences.

This paper offers a first attempt to formalize the Cantillon-Hayek story. Important issues have been ignored to keep the model simple and tractable. Furthermore, only a theoretical link between relative-price effects and economic cycles has been established. These are topics that future scholarship can address.

REFERENCES


APPENDIX A: ALLOCATION OF I-FORM CAPITAL IN THE G SECTOR

Recall from (13) that
\[
\frac{\partial R_t^g}{\partial k_t^{aj}} = \frac{\partial R_t^g}{\partial k_t^{aj}}
\]

From \( R_t^g = x_t^g / p_t \) and (6), it follows that
\[
\frac{\partial R_t^g}{\partial k_t^{aj}} = \frac{v}{p_t} \left[ (k_t^{q_i})^\rho + (k_t^{q_j})^\rho \right]^{\frac{\rho - \rho}{\rho}} \left( k_t^{q_j} \right)^{\rho - 1}
\]

From (15) it follows that
\[
\frac{\partial R_t^a}{\partial k_t^{aj}} = 1
\]

Taken together, we have
\[
\frac{v}{p_t} \left[ (k_t^{q_i})^\rho + (k_t^{q_j})^\rho \right]^{\frac{\rho - \rho}{\rho}} \left( k_t^{q_j} \right)^{\rho - 1} = 1
\]

Rearranging yields
\[
\left( k_t^{q_i} \right)^\rho + (k_t^{q_j})^\rho = \frac{p_t}{v} (k_t^{q_j})^{1 - \rho}
\]

Solving this for \( (k_t^{q_j})^\rho \) yields
\[(k_t^{gj})^\rho = \left(\frac{p_t}{v}\right)^{\frac{\rho}{v-\rho}} \left(\frac{g^{(1-\rho)}}{v-\rho}\right) - \left(k_t^{gi}\right)^\rho\]

Inserting this into (6) yields

\[x_t^g = \left[\left(\frac{p_t}{v}\right)^{\frac{\rho}{v-\rho}} \left(\frac{g^{(1-\rho)}}{v-\rho}\right) - \left(k_t^{gi}\right)^\rho + (k_t^{gi})^\rho\right]^{\frac{v}{\rho}}\]

Simplifying yields

\[x_t^g = \left(\frac{p_t}{v}\right)^{\frac{v}{v-\rho}} \left(\frac{g^{(1-\rho)}}{v-\rho}\right)\]

According to (12), \(k_t^{aj} = 1\). Applying this to (1) implies that \(k_t^{aj} = 1 - k_t^{aj} = 1\). Hence,

\[x_t^g = \left(\frac{p_t}{v}\right)^{\frac{v}{v-\rho}}\]

**APPENDIX B: STEADY STATE**

Inserting \(k_{t+1}^{gi} = k_t^{gi} = \bar{g}^{gi}\) into (17) yields

\[\bar{g}^{gi} = \left(\frac{\bar{g}^{gi}}{v}\right)^{\frac{v}{v-\rho}}\]

Solving for \(\bar{g}^{gi}\) yields (18); that is,

\[\bar{g}^{gi} = (v)^{\frac{v}{\rho}}\]

The steady state relative price results from inserting \(k_t^{gi} = \bar{g}^{gi}\) into (11) and using (12).

**APPENDIX C: CONVERGING CYCLES**

The first-order Taylor approximation of (17) around the steady state value of \(\bar{g}^{gi}\) is

\[\left[k_{t+1}^{gi} - \bar{g}^{gi}\right] = \frac{\partial k_{t+1}^{gi}}{\partial k_t^{gi}} \left[k_t^{gi} - \bar{g}^{gi}\right]\]

with

\[\frac{\partial k_{t+1}^{gi}}{\partial k_t^{gi}} = \frac{v}{v - \rho} \left(\frac{g^{(1-\rho)}}{v}\right)^{\frac{\rho}{v-\rho}} \frac{1}{v}\]
Hence,

\[ k_{t+1}^g - \bar{k}^g = \frac{v}{v-\rho} \left( \frac{k_t^g}{v} \right)^{\frac{\rho}{v-\rho}} \frac{1}{v} \left[ k_t^g - \bar{k}^g \right] \]

Using the steady state value \( k_t^g = \bar{k}^g \) of (18) yields

\[ k_{t+1}^g - \bar{k}^g = \frac{v}{v-\rho} \left( \frac{v}{\rho} \right)^{-\frac{\rho}{v-\rho}} \frac{1}{v} \left[ k_t^g - \bar{k}^g \right] \]

Simplification yields

\[ k_{t+1}^g - \bar{k}^g = \frac{v}{v-\rho} \left[ k_t^g - \bar{k}^g \right] \]

Cycles arise when \( c < 0 \). Because the numerator of \( c \) is nonnegative when \( 0 < \nu \), this condition is satisfied when the denominator of \( c \) is negative. This implies that \( \nu < \rho \).

Cycles are stable when \(-1 < c\), which is satisfied when \( \nu < \frac{\rho}{z} \).