ROTHBARD ON V SHAPED AVERAGE AND TOTAL COST CURVES

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Rothbard (1993, pp. 638–45) refuted the important economic fallacy that excess capacity is a normal consequence of profit maximizing behavior by businesses in some industries when they are in long-run equilibrium. And, in so doing provided a manifest example of misuse of mathematics in modern economics.

According to standard theory, given a U-shaped, average-cost curve (ACC), in equilibrium, a firm whose demand is perfectly competitive will operate at the point where its horizontal demand curve is just tangent to the ACC; i.e., at the point where average cost (AC) is at its minimum. Alternatively, a firm in an industry characterized by monopolistic competition will face a downward-sloping demand curve. In that case, again in equilibrium, the firm will operate where the demand curve is just tangent to the U-shaped ACC. However, in that case, the point of tangency will occur at lesser quantity than that at which AC is at its minimum.

Rothbard, however, puts an end to this notion, despite its vast popularity within the profession. He notes that a necessary condition for the above conclusion is that the ACC be *smooth*. As proof, he offers his famous diagram (1993, p. 644, fig. 72), the essential burden of which is that though the ACC curve slopes downward continuously until it reaches its minimum and then slopes upward continuously, and is in fact a graph of a continuous function, nevertheless the function is not differentiable at critical points, including especially, at its minimum. Our figure 1 accentuates this even more, drawing the ACC not in a "quasi-U" shape, but in a "V" shape. Note that in either of these cases, the downward sloping demand curve can, in contravention of the standard theory, touch[1] the ACC curve at the minimum point of the latter.

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[1]Not "be tangent to." The latter is possible only under the assumption of a smooth ACC.

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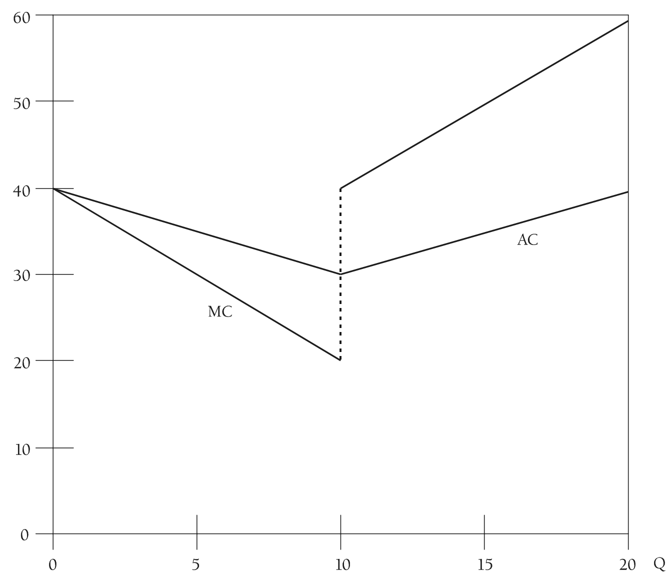
Figure 1



Why, in turn, the universal assumption of smoothness on the part of mainstream economists? This is because of their fetish with differentiation and integration. Unless such tools of economic analysis are used, and this certainly includes but is by no means limited to these two techniques, the feeling is widespread amongst the neoclassicals that there is something woefully wrong. However, human action occurs discretely, not in infinitesimally small steps. Mises (1996, p. 118) states: "There is neither constancy nor continuity in the valuations and in the formation of exchange ratios between various commodities." Thus, the smooth curve assumptions promote mathematical techniques for their own sake, to the denigration of economic considerations. And it is due to the mathematical tail wagging the economic dog, and not the other way around, that we arrive at the fallacy that perfect competition[2] is needed in order to ensure location at the minimum point on the ACC, the fallacy exploded by Rothbard.

So far, so good. However, there are discontinuities, and then there are discontinuities. The difficulty here is with what the ACC implies with regard to marginal cost, and the marginal cost curve (MCC) in our figure 1. To wit, the implication is that there is a discontinuity of gargantuan proportions precisely at the minimum point of the ACC. And this makes no *economic* sense.

We realize full well that Rothbard (1993) is only maintaining his position *arguendo*. He is not at all offering a "quasi-U" shaped, even less our own "V" shaped, ACC, on their own merits. He utilizes them, solely, as a *reductio ad absurdum*.

[2]For other criticisms of the perfectly competitive model, see Barnett, Saliba, and Block (2005).

Nevertheless, fair is fair. It is only proper to acknowledge this shortcoming of the diagram he utilizes to make his point, which we illustrate in our figure 1.

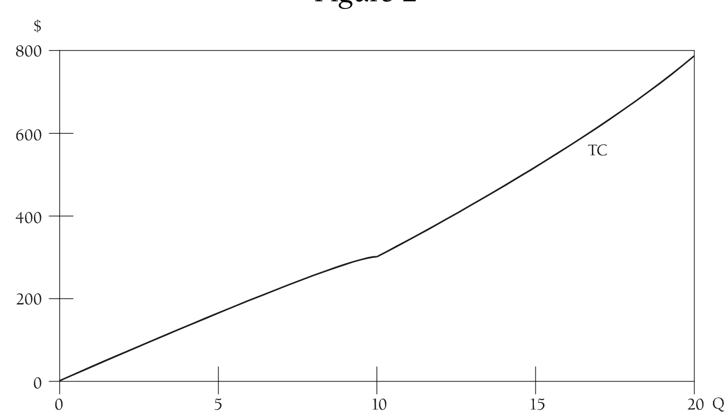
Consider the V-shaped AC curve in figure 1, as per the relevant equation in table 1. It is composed of two straight line segments, one with a negative, and the other with a positive, slope. The AC curve is at its minimum when Q = 10. The relevant MC curve, as per the relevant equation in table 1 lies below the AC curve and decreases continuously up to Q = 10, the Q for which AC is at its minimum. At that exact Q, the MC curve is discontinuous, jumping up above the AC curve and increasing continuously, thereafter. This means that as Q increases from zero units to 10 at which quantity AC is at its minimum: (1) MC is less than AC; (2) MC decreases continuously; and, (3) the (positive) difference between AC and MC becomes continuously greater. Then, when Q hits and continues to increase above that magic quantity, 10, for which AC is at its minimum: (1) MC experiences a discontinuous increase, so that it instantaneously becomes greater than AC; (2) MC increases continuously; and, (3) the (now negative) difference between AC and MC becomes continuously greater.

Table 1

	For Q: $0 \le Q \le 10$	For Q: $10 \le Q \le 20$
Total cost	$40\,Q - Q^2$	$20\,Q + Q^2$
Average cost	$40 - Q$	$20 + Q$
Marginal cost	$40 - 2\,Q$	$20 + 2\,Q$

The equation for the total cost (TC) from which the AC and MC were derived can also be found in table 1, and the TC curve[3] is exhibited in figure 2.

Figure 2



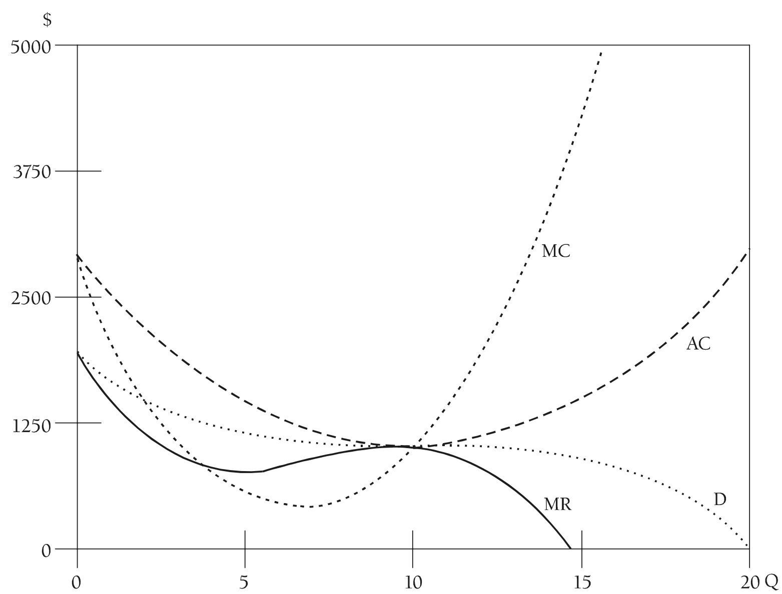
[3]We set the limit at 20 for two reasons, one practical and one theoretical. The practical reason was so that the scaling of the figures would be small enough that the essential features would show up well. The theoretical justification is that given at least one fixed resource, there is an absolute limit to the quantity that can be produced by any single firm; in this case it is 20. At that point all cost curves become perfectly inelastic; i.e., vertical.

This is not to say that a V-shaped average cost curve cannot be drawn, nor that the equation of such a curve cannot be written; i.e., V-shaped cost curves are not erroneous as a matter of mathematics. Rather, they are implausible, to say the least, as a matter of economics.

Rothbard (1993) refuted the standard position by challenging its assumption of U-shaped cost curves by substituting for them a quasi-V-shaped cost curve. In so doing, he implicitly called into question the assumption of differentiability,[4] as manifested in U-shaped cost curves. An alternative refutation, which we now propose, confronts the assumption of linear demand curves. That is, we now substitute a nonlinear demand curve for the traditionally depicted one.

Consider the nonlinear (third-degree polynomial) demand curve in figure 3, as per the relevant equation in table 1.

Figure 3



This demand curve slopes downward continuously except, literally, at one point. The revenue functions were chosen such that, when placed in conjunction with the chosen U-shaped AC cost curve,[5] this one point would occur precisely at the quantity where the AC curve is at its minimum. It can be shown that: (1) for any two points on the demand curve, (P_0, Q_0), and (P_1, Q_1), $Q_1 > Q_0 \Leftrightarrow P_1 < P_0$, that in order to sell a greater quantity the price must

[4]On the importance and use of differentiability in mainstream economics, see Barnett (2003, pp. 57–59).

[5]A glance at the relevant total cost curve, as given in table 2, reveals the absence of any fixed costs; however, this is as it should be, as we are considering long-run curves.

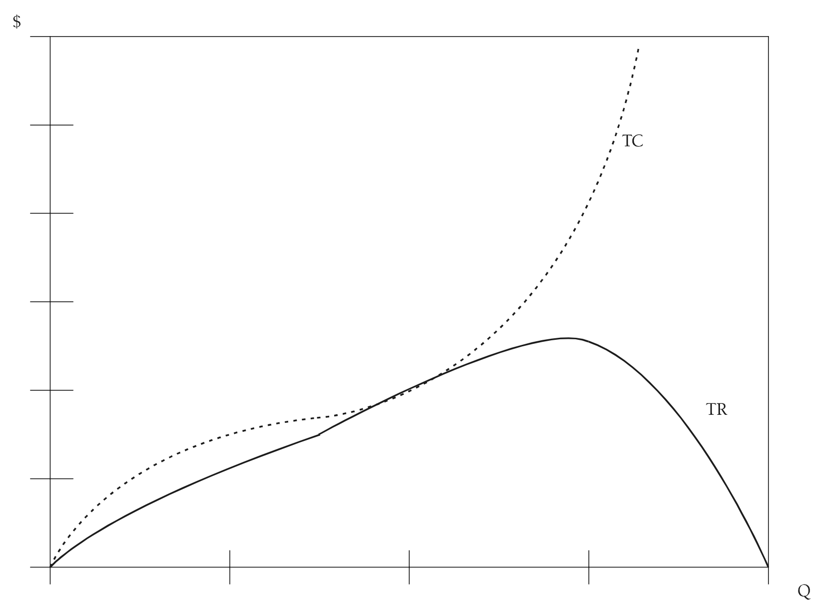
be reduced; and, (2) that for any $Q \neq 10$, $20 (Q - 10)^2 + 1000 > -(Q - 10)^3 + 1000$, that the demand curve lies below the AC curve, save at the one point, Q = 10, at which they are tangent[6] to each other. This, then, is a second case[7] that also refutes the neoclassical doctrine that only in perfect competition can the demand curve be tangent to the AC curve at the curve's minimum point, and therefore that save for perfectly competitive industries, all others exhibit excess capacity in equilibrium.[8]

Table 2

	For Q: $Q \geq 0$
Total revenue	$-Q(Q - 10)^3 + 1000Q$
Average revenue	$-(Q - 10)^3 + 1000$
Marginal revenue	$-3Q(Q - 10)^2 + (Q - 10)^3 + 1000$
Total cost	$20Q(Q - 10)^2 + 1000Q$
Average cost	$20(Q - 10)^2 + 1000$
Marginal cost	$40Q(Q - 10) + 20(Q - 10)^2$

The relevant total revenue and cost curves are in figure 4.

Figure 4



[6]Not, in this case, merely "touching" one another.

[7]We never in a million years would have thought this one up were we not inspired by the critique offered up by Rothbard. We must therefore thank him for blazing this particular path, as he has done so many others.

[8]This is not to say that every nonlinear demand curve will result in a profit-maximizing firm producing at the minimum AC in long-run equilibrium, but rather that certain specific ones would.

And just as Rothbard's (1993) refutation required rejecting standard neoclassical assumptions, so does ours. Where he rejected U-shaped cost curves in favor of quasi-V-shaped ones, we reject demand curves whose associated MR curves decline continuously in favor of those whose MR curves decline, then increase and then decline again. However, it is quite possible, if not even probable, that both Rothbard's cost curves and our demand curves are more realistic than the respective neoclassical counterparts.

Conclusion

It will readily be appreciated that the target here, as was Rothbard's (1993), is the perversion of economics in behalf of mathematical ease and convenience, on the part of mainstream economists. Rothbard (1993) beat them at their own game. He used the very tools of the neoclassical economists to undermine their profoundly mistaken, and, as it happened, anti-free enterprise conclusions. That is to say, he showed that the conclusions reached were the result of trumped up mathematics, not economics. And, by doing so, he further established the Austrian insight concerning the difference in correct methodology between the natural and human sciences; especially regarding the proper use of mathematics. Our contribution has been to emulate Rothbard (1993) by doing to the mainstream what he did to them, only from the demand and marginal revenue side rather from the perspective of costs and supply.

We do claim superiority over Rothbard[9] in one way. As noted above Rothbard refuted the mainstream by substituting a continuous (cost) curve, but one that was not everywhere differentiable, much less twice differentiable, for their twice-differentiable-everywhere curve. To that extent, he did not adhere fully to their "rules of the game." However, we refuted them by substituting a twice-differentiable-everywhere (demand) curve for their straight-line demand curve which, though it is differentiable everywhere, is not twice differentiable anywhere, save in the trivial sense that the derivative of a function that is a constant is zero. Thus we refuted them while adhering fully to their rules of the game.

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[9]This sometimes occurs when an author stands on the shoulders of a predecessor.