DIMENSIONS[1] AND ECONOMICS: SOME PROBLEMS

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> . . . the units of all physical quantities, as well as their magnitudes, [should be included] in all of his calculations.[2] This will be done consistently in the numerical examples throughout the book.
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> –Sears and Zemansky
> *University Physics* 1955

The consistent and correct use of dimensions is essential to scientific work involving mathematics. Their very existence creates the potential for errors: omitting them when they should be included, misusing them when they are included, and others. However, their existence also makes possible dimensional analysis, which can be a significant factor in avoiding error. In the equation $y = f(\cdot)$, if y should have dimensions then so also should

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[1]Throughout, "dimensions" is used generically and "units," specifically. Thus, distance is a dimension and centimeters, meters, and feet are among the alternative units of the distance dimension.

[2]Sears and Zemansky (1955, p. 3) distinguish units and magnitudes, magnitudes being pure numbers, as follows:

> We shall adopt the convention that an algebraic symbol representing a physical quantity, such as F, p, or v, stands for both a *number* and a *unit*. For example, F might represent a force of 10 lb, p a pressure of 15 lb/ft^2, and v a velocity of 15 ft/sec. When we write $x = v_0 t + \frac{1}{2} at^2$, if x is in feet then the terms $v_0 t$ and $\frac{1}{2} at^2$ must be in feet also. Suppose t is in seconds. Then the units of v_0 must be ft/sec and those of a must be ft/sec^2. (The factor ½ is a *pure number*, without units.)

f, and they should be identical to those of *y*. If *y* should not have them then neither should *f* have them. Such an analysis of $y = f(\cdot)$ would determine: (1) which, if any, dimensions *y* and each element of *f*, and consequently *f*, itself, should have; and, (2) whether the dimensions of *f* and *y* are identical, which is a necessary, though not sufficient, condition for the equation to be correct. An error revealed by a correctly performed dimensional analysis indicates a fundamental problem.[3] Therefore, the importance of dimensions for science can hardly be overstated.

The first sections of this paper consider, respectively, the following two problems that arise when dimensions are not correctly included in economic models: (1) those that are meaningless or economically unreasonable; and, (2) those that are inconstant—i.e., the same constant or variable having different dimensions, as if velocity were sometimes measured in meters per second and other times measured in meters only or in meters squared per second.[4] The third section provides a macroeconomic example of the "dimensions problem" from an article in a recent issue of a leading English language economics journal. Section four contains a discussion; and the final section, the conclusions.

The analysis in this paper concerns production functions and is robust with respect to increases in the number of independent variables and to alternative functional forms.[5] Moreover, the analysis is robust with respect to others used in economic theory: e.g., utility, demand, and supply functions.

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[3]Sears, Zemansky, and Young state:

> When a problem requires calculations using numbers with units, the numbers should always be written with the correct units, and the units should be carried through the calculation as in the example above. This provides a useful check for calculations. *If at some stage in the calculation you find an equation or expression has inconsistent units, you know you have made an error somewhere.* In this book we will always carry units through all calculations, and we strongly urge you to follow this practice when you solve problems. (1987, p. 7; emphasis added)

"Dimensional analysis is used to check mathematical relations for consistency of their dimensions . . . *[i]f the dimensions are not the same, the relation is incorrect.*" (Cutnell and Johnson 2001, p. 6; emphasis added)

[4]These are *not* those of aggregation in disguise; they can, and do, exist in models of but one good and one resource, labor. Rather, the issues dealt with here are even more basic and devastating for mathematical economics and econometrics than is that of aggregation.

[5]Although not directly related to the subject under discussion, it should be noted that there is a fundamental problem with the use of the mathematics of functions in economics. One *sine qua non* of a function is that, for any specific set of values of the independent variables, there must be a *unique* value of the dependent variable.

"Let X and Y be nonempty sets. Let f be a collection of ordered pairs (x, y) with $x \in X$ and $y \in Y$. Then f is a function from X to Y if to every $x \in X$ there is assigned a unique $y \in Y$" (Thomas 1968, p. 13; emphasis added).

Meaningless or Economically Unreasonable Dimensions

One widely used function[6] is a 2-input "Cobb-Douglas" (CD) production function. A typical CD function is given by $Q = AK^{\alpha}L^{\beta}$, in which: Q is the output variable; K and L are the capital and labor input variables, respectively; A, may be a constant or a variable; and, α and β are the elasticity of output with respect to capital and with respect to labor, respectively. Consider a 2-input, CD, production function for a specific good, widgets: $Q = AK^{\alpha}L^{\beta}$. If dimensions are used correctly, output, capital, and labor each must have both magnitude *and dimension(s)*, while α and β are pure numbers. Assume, for example,that:[7]

(1) Q is measured in *widgets/elapsed time (wid/yr)*;
(2) K is measured in units of *machine-hours/elapsed time (caphr/yr)*; and,
(3) L is measured in *man-hours/elapsed time (manhr/yr)*.

Then a dimensional analysis of the production function $Q = AK^{\alpha}L^{\beta}$ establishes that A (= $Q/K^{\alpha}L^{\beta}$) is measured in (*widgets/elapsed time*)/([*machine-hours/elapsedtime*]$^{\alpha}$ • [*man-hours/elapsed time*]$^{\beta}$); i.e., in (*wid* • $yr^{(\alpha+\beta-1)}$)/(*caphr*$^{\alpha}$ • *manhr*$^{\beta}$).

Therefore, it is incorrect to express production relationships in any case in which Leibensteinian style X-inefficiency can exist. For an example of such a situation see note 11.

[6]This is an understatement. It is probably no exaggeration to claim that the CD is the most widely used mathematical example in all of neoclassical economics.

[7]Because there are no standard systems of dimensions or units in economics, specific, but nonstandard, units are used. It should be noted that, so long as matters are confined to mathematical models, the issue of dimensions/units can simply, though indefensibly, be ignored, this is no longer true when the matter turns to the estimation of econometric models. Then, data must be used. If every variable is measured in monetary terms, the problem of dimensions does not arise. Of course, measuring every variable in monetary terms raises other problems. For example, although some input variables may be, and sometimes are, measured in terms of nonvalue (i.e., "real") units (e.g., of man-hours for labor input), the input of capital goods is invariably measured in value (i.e., monetary) units, and the output is virtually always measured in monetary units. On the one hand, this raises the aggregation problem *re* heterogeneous capital goods; on the other it presents the difficulty of the circularity of the measurement of the value of the capital because of the role of the interest rate in determining the present value of a quantity of capital goods and the role of the quantity of capital goods in determining the interest rate. On these points see Harcourt (1972, pp.1-46).

Moreover, if real units are used, then production functions are consistent with economic theory in that particular quantities of the various inputs combine to produce a specific quantity of the output. However, if monetary units are used such production functions set economic theory on its head, for then particular values of the various inputs combine to produce a specific value of the output. But, economic theory teaches that the value of the inputs is derived from the value of the output. (Thanks to an anonymous referee calling attention to this omission in the prior submission.)

Only positive values of α and of β are acceptable, as nonpositive values for either, or for both, imply negative or zero marginal productivity of the relevant input(s). If $\alpha = \beta = 1$, then the dimensions of K^α, L^β, and Q—machine-hours per year, man-hours per year, and widgets per year, respectively—are meaningful. But, the dimensions of A are widgets per (machine-hours • man-hours) per year or wid/(caphr • manhr)/yr. For those dimensions to be meaningful, requires, at a minimum, that the product of machine-hours and man-hours is meaningful, a dubious proposition indeed. However, even if the dimensions are meaningful in this case, they are economically unreasonable. For, if $\alpha = \beta = 1$, the marginal products of both K and L are positive constants (the Law of Diminishing Returns is violated) and there are unreasonably large economies of scale—a doubling of both inputs, *ceteris paribus*, would quadruple output.

Alternatively, if it is not true that $\alpha = \beta = 1$, then either α or β, or both, have noninteger values or integer values of two or greater. Noninteger values of α or β, or both, result in such units as, for example, $(\text{man-hours/year})^{0.5}$ or $(\text{man-hours/year})^{1.5}$ for L^β, and similarly for K^α. But the square roots of man-hours and of years are meaningless concepts, as are the square roots of the cube of man-hours and the cube of years. Also, integer values of two or greater for α or β, or both, result in such units as, for example, $(\text{man-hours/year})^2$ or $(\text{man-hours/year})^3$ for L^β, and similarly for K^α. But the squares of man-hours and of years are meaningless concepts, as are the cubes of man-hours and of years, and similarly for machine-hours. (The units of A are even more meaningless, if that is possible.) Therefore, no matter what the values of α and β, the dimensions are either meaningless or economically unreasonable.

If the same 2-input, CD, production function, $Q = AK^\alpha L^\beta$, is used, but Q is taken to be aggregate output, then the function is an aggregate, or macroeconomic, production function. However, and for the same reasons as in the microeconomic example, a correct use of dimensions here also yields dimensions that are either meaningless or economically unreasonable. Moreover, an additional problem, that of aggregation, arises in the macroeconomic case.

The problem of dimensions that are either meaningless or economically unreasonable cannot be eliminated by using more complex production functions such as the constant elasticity of substitution (CES); if anything, it is exacerbated.

A correct use of dimensions in these examples, then, yields results that are either meaningless or economically unreasonable. However, these problems only become evident when dimensions are correctly included in the model, which is rarely[8] the case with economic modeling.

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[8]The only cases the author is aware of are a few instances involving Fisher's equation of exchange.

INCONSTANT DIMENSIONS

To reiterate, this problem consists in the same constant or variable having different dimensions, as if velocity were sometimes measured in meters per second and other times measured in meters only or in meters squared per second. It can be illustrated by comparing (Newtonian) gravity[9] with production functions. Gravitation is a force. A force *(F)* exerted on a body may be measured as the product of its mass (m) times its acceleration (a);[10] i.e., $F = m \cdot a$. In the meter-kilogram-second (mks) system the units of F are kilograms•meters /(second^2); i.e., kg•m/(sec^2). Sir Isaac Newton's law of universal gravitation, may be stated:

> Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of the masses of the particles [m and m'] and inversely proportional to the square of the distances between them [r^2].
>
> $$F \propto mm'/r^2.$$
>
> The proportion above may be converted to an equation on multiplication by a constant G which is called the gravitational constant:
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> $$F = G \cdot (mm'/r^2).$$ (Sears and Zemansky 1955, p. 79)

Then: $G = F/(mm'/r^2)$. And, logical and physical consistency require that G have the dimensions of $F/(mm'/r^2)$. Using the mks system $F/(mm'/r^2)$ has the units (m^3)/(kg•sec^2); therefore, G must have the units (m^3)/(kg•sec^2).

This result has been invariant for countless measurements of G over the past three centuries: regardless of the magnitude, the dimensions have always been distance^3/mass•(elapsed time)^2; e.g., m^3/(kg•sec^2) in the mks system.

Unfortunately, such is not the case in economics. Compare that result—the constancy of the dimensions—with the results of measurements of a 2-input, CD production function. Such measurements yield estimates for α, β and A. Invariably, alternative estimates of α, β, and A differ. This is not surprising, but it does present a serious problem. Because A has both magnitude and dimensions, different values of α and β imply different dimensions for A, such that, even though the dimensions in which Q, K, and L are measured are constant, the dimensions of A are inconstant. For example, let Q, K, and L be measured in the same units as in the section "Meaningless or Economically

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[9]Although in this paper the analysis involves Newtonian gravity, the results of the analysis are robust for all applications in the natural sciences.

[10]It is true that a force is a vector quantity; i.e., it has a directional quality, as well as a magnitude (Sears, Zemansky, and Young 1987, p. 10). However, this is irrelevant for this analysis.

Unreasonable Dimensions." Then, if the magnitudes of α and β are measured as 0.5 and 0.5, respectively, then the units of A are wid/(manhr$^{0.5}$ • caphr$^{0.5}$). However, if the magnitudes of α and β are measured as 0.75 and 0.75, respectively, then the units of A are (wid • yr$^{0.5}$)/(manhr$^{0.75}$ • caphr$^{0.75}$).

The problem of inconstant dimensions (or economically unreasonable results) cannot be eliminated by using more complex production functions such as the CES; if anything, it is exacerbated.

A correct use of dimensions in this example, then, yields inconstant dimensions. Inconstant dimensions are, of course, a nonsensical result. However, this problem only becomes evident when dimensions are correctly included in the model, which is rarely the case with economic modeling.

Macroeconomic Example

Consider the following, from a model in a recent issue of a leading Englis-language economics journal.

1. In the section on households, the "[f]unction *H* measures the disutility from work, which depends on hours (*N*) and effort (*U*)." The arguments in the utility function of the representative household include $\Sigma\beta^{t}H(N_t, U_t)$; $t = 0 \ldots \infty$, where "$\beta \in [0, 1]$ is the discount factor" and *t* is the index of the time period.[11]

2. The section on firms posits that,

> [t]here is a continuum of firms distributed equally on the unit interval. Each firm is indexed by $i \in [0, 1]$ and produces a differentiated good with a technology $Yit = ZtLit^{\alpha}$. Li may be interpreted as the quantity of effective labor input used by the firm, which is a function of hours and effort: $Lit = Nit^{\theta}Uit^{1-\theta}$ where q ∈ [0, 1]. Z is an aggregate technology index, whose growth rate is assumed to follow an independently and identically distributed (i.i.d.) process $\{\eta_t\}$, with $\eta_t \sim N(o, s_z^2)$. Formally, $Z_t = Z_{t-1} \exp(\eta_t)$.[12]

3. The section on equilibrium maintains that,

> [i]n a symmetric equilibrium all firms will set the same price P_t and choose identical output, hours, and effort levels Y_t, N_t, U_t. Goods market clearing requires . . . Y_{it} Y_t , for all $i \in [0, 1]$, and all *t*.

[11]Obviously, the time period index, *t*, was inadvertently omitted from the consumption variable in the representative household's utility function.

[12]Because effective labor, L_{it}, is an argument in the production function of output, Y_{it}, and because effective labor is a function of the level of effort, U_{it}, Leibensteinian style X-inefficiency can exist in this model. On the importance of this, see note 5.

Furthermore, the model yields "the following reduced-form equilibrium relationship between output and employment: $Y_t = AZ_tN_t^{\varphi}$."

Among the conclusions that can be drawn from this model, each of which will be considered in turn, are: (1) the number of firms and the number of households is identical, and is equal to infinity; (2) the quantity of each input used by each firm is identical to the quantity of each input provided by each household; and, (3) there are an infinite number of *differentiated* goods, each of which is *identical* to every other good.

First, the continuum of firms necessarily means that there is an infinite number thereof.[13] Assume, *arguendo*, that the (infinite) number of firms is given by n. Then, as each firm uses the same number of hours, N_t, and the same effort level, U_t, as every other firm, the total hours used are nN_t and the total effort level is nU_t. However, because N_t and U_t also are the hours and effort level of the representative household, unless there are exactly n households providing nN_t total hours and nU_t total level of effort, either the firms are using more hours than the households are actually working, or they are using less. The same can be said for the level of effort. Only if the number of households is n are the number of hours used and the level of effort used exactly equal to the number of hours worked and the level of effort provided. Of course, this would necessarily mean that there is an infinite number, n, of households exactly equal to the infinite number, n, of firms.

Second, because there would be one (identical but for the nature of the output) firm per (identical) household, each firm would use exactly the hours and effort level put forth by one of the households, though, conceivably, the hours and level of effort used by a particular firm would not all come from the same household.

Third, because $Y_t = AZ_tN_t^{\varphi}$,[14] and A and Z_t are both dimensionless magnitudes,[15] Y_t must have the same dimensions as N_t^{φ}. The dimension of N_t is

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[13]"A set forms a continuum if it is infinite and everywhere continuous, as the set of reals or the set of points on a line interval" (Glenn and Littler 1984, p. 37).

[14]It could be argued that, because Y_t is the output of a single firm, $Y_t = AZ_tN_t\varphi$ is not an example of a macroeconomic production function. However, because there are n identical (but for their differentiated goods) firms, the aggregate production function is $nY_t = nAZ_tN_t$. The microeconomic and macroeconomic functions, then are identical up to a linear scaling factor, n. That the firms' goods are differentiated does not prevent us from aggregating them in this model because, as is shown in the text, the differentiated goods are not differentiated at all; rather, they are identical. (If dimensions were being used, n and Y_t would have dimensions; e.g. firms, and widgets per firm in time period T, respectively. In that case, nY_t and Y_t would have different dimensions: widgets in time period T, and widgets per firm in time period T, respectively. However, because each firm's output is identical to every other firms' output, we could still validly aggregate their outputs by multiplying Y_t by n.)

[15]We are given that: $A \equiv [\lambda_n(1\text{-}\theta)/\lambda_u\theta]^{\alpha(1\text{-}\theta)/(1+\sigma u)}$; $\theta \in (0, 1)$ and is, therefore, dimensionless; and, λ_n, λ_u, σ_n, and σ_u are positive constants. We know that λ_n, λ_u, σ_n, and σ_u

hours (hrs); and φ is a positive, dimensionless, constant.[16] Therefore, the dimensions of Y_t are hrs$^{\varphi}$. In any case in which $\varphi \neq 1$, the dimension of Y_t, (hrs)$^{\varphi \neq 1}$ is meaningless; e.g., (hrs)$^{0.5}$, (hrs)$^{1.5}$, and (hrs)^2 are meaningless dimensions.[17] Alternatively, if $\varphi = 1$, then $Y_t = AZ_tN_t$, and the dimension of Y_t is the same as that of N_t, hrs. However, in that case, because $Y_t = AZ_tN_t$, the output hours are less than, equal to, or greater than, the input hours as AZ_t is less than, equal to, or greater than one (1). But if output is measured in hours, then the output hours cannot be greater than or less than the input hours; rather, the output hours must be equal to the input hours; i.e., $AZ_t \equiv 1$ and $Y_t \equiv N_t$. Therefore, each and every firm uses input of exactly N_t hrs to produce exactly N_t hrs of output; i.e., there is no net production—not one of the infinite number of supposedly profit maximizing firms produces more hours of output than the number of hours it uses as input.

Moreover, because $Y_{it} = Y_t \; \forall \; i$, the dimension of every firm's output is hrs. Therefore, each of the n differentiated goods produced by the n firms consists of homogeneous hours.

Surely, this model is not defensible.

Discussion

The problems caused by the failure to use dimensions consistently and correctly in production functions—dimensions that are either meaningless, unreasonable, or inconstant—are not minor problems, and by no means are restricted to production functions. Rather, these problems are both critical and ubiquitous—they afflict virtually all mathematical and econometric models of economic activity. And that, unfortunately, is the way modern economics is done (Leoni and Frola 1977; and Mises 1977).

A more or less standard pattern can be discerned in articles in mainstream economics journals. First, the gist of a theory is concisely developed. Second, a more or less complex mathematical model of the theory is elaborated and solved. Third, an econometric model based thereon is constructed, and estimates of the magnitudes of the parameters and of the relevant statistics are

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are dimensionless from the context in which they first appear: H(Nt, Ut) = $(\lambda_n N_t^{1+\sigma n}/(1+\sigma_n)) + (\lambda_u N_t^{1+\sigma u}/(1+\sigma_u))$. And, we know that α is a positive, dimensionless, constant from the context in which it first appears: $Y_{it} = Z_{it}L_{it}^{\alpha}$. Therefore A must be a positive dimensionless constant. We also know that Z_t must be a positive, dimensionless, variable because, "Z is an aggregate technology index, whose growth rate is assumed to follow an independently and identically distributed (i.i.d.) process $\{\eta_t\}$, with $\eta_t \sim N(o, s_z^2)$. Formally, $Z_t = Z_{t-1} \exp(\eta_t)$."

[16]We are given that: $\varphi = \alpha\theta + \alpha(1\text{-}\theta)(1+\sigma_n)/(1+\sigma_u)$. We know that $\theta \in (0, 1)$, σ_n, σ_u, and α are positive, dimensionless, constants. (See note 3.) Therefore, φ must be a positive, dimensionless constant.

[17]It is true that time (t) squared does have a meaning in the world of the natural sciences, such that the dimensions of acceleration are distance/t^2. But that does not in any way help us find meaning for t^2 in the world of the social sciences.

provided. Fourth, there is an explanation and discussion of the empirical results. Fifth, conclusions are drawn. Sometimes some of the mathematical manipulations may be relegated to an appendix if they are considered too abstruse for the body of the paper.

This methodology entails generating hypotheses or retrospective predictions, based on the theory, about the magnitudes of the relevant parameters of the model. Then, using the techniques of statistical inference, the estimated signs and magnitudes of the parameters are compared with their expected signs and magnitudes, respectively, to determine if the hypotheses may be falsified or the retrospective predictions rejected as insufficiently accurate.

Unfortunately, the failure to use dimensions consistently and correctly makes it almost impossible to prevent untenable and unreasonable assumptions from entering into the mathematical and econometric models undetected. Such assumptions, of course, render the models so afflicted virtually worthless. They make possible, as we have seen, such indefensible results as differentiated goods that are identical. (Or, to amend slightly a remark of Coase (1988, p. 185), "In my youth it was said that what was too silly to be said may be sung. In modern economics it may be put into [dimensionless] mathematics.") Such clearly untenable results go unchallenged because the dimensionless mathematics obfuscate, rather than illuminate, the analysis and also because some are intimidated by the mathematics.

Certainly, the problems exposed in the examples could have been avoided had dimensions been used consistently and correctly. Whether, then, the author could have developed a tractable model is a different matter. Nevertheless, it is clear, at least to the present author, that anytime the choice is between a dimensionless, tractable, mathematical model or none at all, the latter is by far the better choice.

None of this should be taken to say that the author whose work provided the example did not have valuable insights into the economic activities with which he was concerned—he well may have. However, that must be determined independently of his mathematical model, as it provides no valid support for his argument.

CONCLUSIONS

The economics profession has attempted to achieve the degree of success in understanding, explaining, and predicting events in the social world that physicists and engineers have achieved in the natural world by emulating their methods; i.e., using mathematical and statistical analyses to model, understand, and explain, the relevant phenomena. However, in so doing, economists have failed to emulate physicists and engineers in one essential aspect of their work: the consistent and correct use of dimensions. This is an abuse of mathematical/scientific methods. Such abuse invalidates the results of mathematical and statistical methods applied to the development and application of economic theory.

Neither is this problem a thing of the past, nor is it one confined to lesser or fringe venues. Rather, it is a continuing problem and one found in the leading mainstream journals (and textbooks). Because young minds are formed by such materials, future generations of economists are being brought along in a faulty tradition. And, unless and until this changes, and economists consistently and correctly use dimensions in economics, if such is possible, mathematical economics, and its empirical alter ego, econometrics, will continue to be academic games and "rigorous" pseudosciences. However, if for no other reason than the influence of the economics profession on governmental policies, such games and pseudosciences are not without their costs in the real world.

This is not to say that there have not been advances in economic understanding by the neoclassicals, but rather to argue that mathematics is neither a necessary nor a sufficient means to such advances. Whether it even is, or can be, a valid means to such advances is a different issue. What is certain, however, is that mathematics cannot possibly be a valid means unless and until it is used properly. Among other things, that means that dimensions must be used consistently and correctly.

ADDENDUM

There is an ongoing debate in the literature as to whether Austrian School economists should attempt to publish in mainstream journals or rather in nonmainstream journals created specifically for the purpose of providing a venue for explicitly Austrian work. Among the most recent additions to this literature are Rosen (1997); Yeager (1997, 2000); Vedder and Gallaway (2000); Laband and Tollison (2000); Backhouse (2000); Block (2000); and Anderson (2000).

One issue centers about type one errors; i.e., the exclusion of explicitly Austrian work, regardless of quality, from mainstream journals, and, *a fortiori*, top-tier, mainstream journals. As such, it is implicitly assumed that Austrians should aspire to publish in such journals.

Perhaps of more importance is the issue of whether Austrians should aim to publish in such journals. This raises the spectre of type two errors; i.e., the inclusion in such journals of material that should have been excluded for lack of quality.

Yeager (1997, pp. 159-64; 2000) attacks the concept of the so-called "marketplace for ideas." The marketplace for ideas in economics is taken to constitute the top-tier mainstream journals. Whereas, the exclusion of Austrian work from these journals is taken to mean that Austrian ideas have failed the test of the market, Yeager points out the perversities of such a test. He argues that Austrian economics is not valueless merely because it is uncompetitive in that market and thus, implicitly, Yeager would agree that there should be a venue for good Austrian work that cannot be published therein. However, he does not claim that such journals have no value, and can be interpreted as saying that Austrians should publish in top-tier, mainstream journals when possible and in specifically Austrian venues only as a fallback position. Vedder and Gallaway (2000) also can be reasonably read to arrive at the same conclusion.

Block (2000, p. 55), presumably on the grounds of type two, as well as type one, errors challenges the very legitimacy of the editors of mainstream journals: "One difficulty is that Vedder and Gallaway *unnecessarily concede* to the very editors they accuse of bias against Austrians *a modicum of legitimacy*" (emphasis added). Anderson (2000) makes explicit the argument that these journals commit many and serious type two errors. Whether he would also prefer the strategy of publishing in specifically Austrian journals as a fallback is not clear. However, Block (2000, pp. 55–56) states that in "[i]n [his] view, the leading economic journals are the Austrian ones."

Most of the debate in the literature cited above concerns anti-Austrian bias of the editors and referees of mainstream journals. *In what follows I question the competence of the editors and referees of top-tier, mainstream journals on other grounds and, therefore, the desirability of attempting to publish in them.* Specifically, I challenge the competence in mathematics of these editors and referees and make the case by relating a real life case. The facts, as revealed in the referees' reports (on a paper submitted for consideration for publication), the response thereto, and the co-editor's follow-up correspondence, prove beyond any doubt that the referees and co-editor were incompetent to judge the paper. And this is not a case of opinion, theirs versus mine; no, it is a clear and indubitable case of their commission of mathematical errors.

One requirement for the proper use of mathematics is the correct use of dimensions/units.[18] For example, dimensional analysis is used in physics and engineering to insure the consistency of the relationships in an equation. The economic variables one sees in the mathematical and statistical models ubiquitous in economics (Backhouse 2000) always involve dimensions. However, dimensions/units are rarely used in economics, and dimensional analysis virtually never. Consequently, I submitted a paper,[19] on this subject to a leading English language economics journal. The paper was an indirect attack on the use of mathematics in economic theory. It maintained that if one uses mathematics in economics one must do so correctly.[20] It demonstrated that a dimensional analysis of production functions, specifically, the Cobb-Douglas,

[18]This is not a difficult thing, and, in fact, there is published work on the subject of dimensional analysis. For an example, see the appendix in Reddick and Miller (1955).

[19]That paper, with minor, nonsubstantive editorial changes, constitutes the body of the present article, save that, at the suggestion of a referee, one (1) example was removed, solely because it involved the Leontief, fixed-coefficient, production function that the referee thought to be "of little general interest and of no interest to readers of the *QJAE*."

[20]Although Austrians should only use mathematics when doing history (Block 2000, p. 48), when they do use it this dictum applies to them as well.

yields meaningless or economically unreasonable, and inconstant, dimensions. It then provided two examples of the consequences of the failure to use dimensions and dimensional analysis in economics, one microeconomic and one macroeconomic, criticizing articles in then-very-recent issues of that same journal. Application of dimensional analysis resulted in the conclusion that both models were untenable and nonsensical.[21]

The paper was rejected. Included with the letter of rejection were the reports of three referees. The following are excerpts from these reports along with brief proofs of the errors therein.

> From referee #1's report:
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> A "defect" in economic analysis is proposed, in that equations do not properly account for units, and that two sides of equations used generally in economics are therefore inconsistent. It is claimed that this defect is not present in the physical sciences, such as physics, and that this defect invalidates most formal economic modeling. The "defect" is best illustrated by an example taken from the paper, which I shall detail next. I shall then show that this "defect" is also present in physics by using illustrations from a random book off of my shelf that has some examples of simple physical systems. Then I shall argue that this is, in fact, not a defect at all.
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> From referee #2's report:
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> Dimensional analysis can only be applied to *laws*.
>
> A case in which this [dimensional] analysis made sense in economics was its application to Fisher's relation of exchange: $MV = PT$. This is one of the few examples in economics that comes closest to a law. One result of dimensional analysis is that there is something odd with this equation. The left part does contain a time dimension, while the right side doesn't. This is not something new and can be found in any textbook.
>
> And, from referee #3's report:
>
> There is no question that the lack of dimensional consistency is pervasive throughout mathematical economics. However, this paper does not make clear why this lack of dimensional consistency is problematical. The lack of dimensional consistency is not so much a problem in and of itself . . .

Compare the referees' statements with the following taken from two leading (basic) physics textbooks.

> Dimensional analysis is used to check mathematical relations for the consistency of their dimensions . . . *[i]f the dimensions are not the same, the relation is incorrect.* (Cutnell and Johnson 2001, p. 6; emphasis added)

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[21]This is not to say that the authors of the articles might not have had something else of value to say. That is another issue.

> An equation must *always* be dimensionally consistent; this means that two terms may be added or equated only if they have the same units. . . . When a problem requires calculations using numbers with units, the numbers should *always* be written with the correct units, and the units should be carried through the calculation as in the example above. This provides a useful check for calculations. *If at some stage in the calculation you find that an equation or expression has inconsistent units, you know you have made an error somewhere*. (Sears, Zemansky, and Young 1987, p. 7; emphasis added)

Is it possible to believe that anyone with even the most elementary training in mathematics could make the statements made by these referees? This is incredible! Are the referees innumerate? How else to explain the foregoing? But there is more.

> Also from referee #1's report:
>
> The details are not very important, but the solution to the problem [of simple harmonic motion] posed [Spiegel 1967, p. 186] is $x = 1/3 \cos 8t$, where x is distance measured in feet (the deviation from the equilibrium position of the weight) and t is time measured in seconds. So exactly what kind of conversion constant [sic] do you [Barnett] want to use to convert time into [sic] distance? It is evidently not a constant, since it must be passed through the cosine expression [sic] (similar to passing units of labor or capital through the exponents [in $Q = AK^{\alpha}L^{\beta}$] above.)

But of course the details are important, because for this referee the devil is in the details. A formula, $x = A \cdot \cos \omega t$, for the displacement in simple harmonic motion can be found in Cutnell and Johnson (2001, p. 278). In this formula: x is the displacement, measured in units of length; A is the amplitude of the simple harmonic motion, also measured in units of length; ω is the constant angular speed, measured in radians/second (rad/sec); and, t is the elapsed time, measured in seconds (sec). Consequently, ωt has the dimensions rad.

Restating the formula, $x = A \cdot \cos \omega t$, with the appropriate units attached, and using feet (ft) as the unit of length, yields: x[ft] = A[ft] $\cdot$ cos ω[rad/sec] $\cdot$ t[sec]. Canceling the sec on the right-hand side yields: x[ft] = A[ft] $\cdot$ cos ω[rad]t.

However, a radian is a dimensionless measure of a plane angle[22] (1 rad = $180°/\pi \approx 57.30°$). Therefore, the only unit that "must be passed through the cosine expression" is an (plane) angular measure. Of course, converting plane angular measures, whether radians or degrees, to a pure, i.e., dimensionless, number is precisely what the trigonometric operators, cosine included, do. Consequently, the equation $x = A \cdot \cos \omega t$ has the unit feet on both sides.

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[22]Http://physics.nist.gov/cuu/Units/units.html.

Compare the referee's specific equation, $x = 1/3 \cos 8t$, with the generic form, $x = A \cdot \cos \omega t$. The correspondences between the terms in these equations are: $x = x$; $A = 1/3$; magnitude of $\omega = 8$; and, $t = t$. Use these correspondences to restate the equation, $x = 1/3 \cos 8t$, with the appropriate units made explicit, as: x[ft] = 1/3[ft] cos (8[rad/sec] $\cdot$ t[sec]). Cancel the sec on the right-hand side to obtain: x[ft] = 1/3[ft] cos 8[rad] $\cdot$ t, where the term cos 8[rad] $\cdot$ t is dimensionless. Then, as must be the case, the units on the right-hand side are identical to the units on the left-hand side; to wit, in this case, feet. Obviously, the referee's statement is erroneous.

This example brings to mind the term *idiot savant*. No doubt, this referee knows a great deal of pure mathematics; but, does he know anything at all about applied mathematics or physics? On the evidence he provides in the foregoing excerpt, the answer, at least with respect to harmonic motion, is a resounding, "NO!"

And, yet again, from the same referee #1:

> If one wants an example from physics not involving time, try p. 97 [Spiegel 1967], where there is an example concerning thermal conductivity in pipes. The solution is $U = 699 - 216 \ln(r)$, where r is distance in centimeters and U is temperature in degrees. Now what kind of conversion factor do you want to use to convert distance into degrees? I conclude that physics contains the same "defect" when certain systems are examined.

Once again, this referee exhibits his ignorance of applied mathematics and physics, at least with respect to thermodynamics. In fact the dimensions on *both* sides of the equation $U = 699 - 216 \ln(r)$ are degrees centigrade. As the proof is somewhat lengthy it is included as an appendix.

The following, with emphasis added, is the corpus of the co-editor's letter of rejection that accompanied the referees' reports excerpted, above.

> I enclose three *thoughtful* reports on your manuscript. The referees, while sympathetic, unambiguously recommend rejection. *I agree with these assessments* and must reject your manuscript.
>
> The referees on occasion adopt a somewhat harsh tone. I hope you can see that they took the refereeing responsibility very seriously and have written *thoughtful* reports. They labored to understand your thinking, and the occasional harsh word is the consequence of frustration, one that I felt in reviewing your manuscript as well.
>
> The [journal] receives about 1000 manuscripts per year, and publishes less than ten percent of these. As a consequence, I am forced to reject many quite good manuscripts. Thank you for submitting your paper to the [journal]. I am sorry my response could not be more satisfying.

I submitted a *12-page* reply to the referees' reports in which numerous errors were called to the co-editor's attention, specifying, for each error, the nature thereof, and providing, for each, a detailed proof of the error.

The co-editor responded to my reply with a letter dated February 1, 2001, the corpus of which follows.

> I am responding to your letter of Jan 12, 2001.
>
> Evidently you could not see past the tone of the reports to the substance of the reports. Unfortunately, reading *your diatribe on the referees' errors has not convinced me of the error of their ways.* The case of the Cobb-Douglas production function is quite clear. Just because you think that the units associated with C-D are unnatural doesn't make it so. *Moreover, the referees are right about the units required to rationalize physics*—distanced squared or log(temperature) makes no more sense than the square root of manhours. The units are what they are, and certainly you can't really think the Cobb-Douglas production function is logically inconsistent. Like a law of nature, a production function is whatever it is.
>
> As you surmised, I am not going to reopen the file. *At the very minimum, you have failed to convince three referees and one editor of the merit of your approach.* A great deal more effort into communicating the results is going to be necessary, I suspect, to sell this work to any journal. You could try *Economics & Philosophy.*

The editor clearly states that he "agree[d] with these [referees'] assessments," that the reports were "thoughtful," that my "diatribe" did not "convince [the editor] of the error of [the referees'] ways," and that, "[a]t the very minimum, you have failed to convince three referees and one editor of the merit of your approach." My reply incorporated the material included above and in the appendix; moreover, it went into greater detail. How, then, could the editor reach the conclusions he did? Three referees and an editor at a top-tier journal and all innumerate? Never in my wildest dreams did I think that an editor and referees for one of the most prestigious English-language economics journals could be so ignorant in a matter of basic mathematics, much less that they would commit such to paper, where it cannot be denied and can, and is being preserved for posterity.

This brings me back to the basic issue, the desirability of specifically Austrian journals. Given the extent of the mathematization of economics, it is critically important that mathematics, if used at all, be used correctly in economics. Therefore, the subject matter of my original paper is very important. Moreover, if my paper is correct—if, in fact, mathematics is abused/misused in economics if for no other reason than that articles in at least one top-tier journal cannot pass the test of dimensional analysis—the paper is worthy of publication in an important venue. The fact that it was rejected by referees and an editor incompetent to the task because of a demonstrated lack of understanding of the basic mathematics of dimensions, the very subject matter of the paper, provides a sufficient reason to have alternative outlets, i.e., specifically Austrian journals, available.[23] Moreover, I think it highly improbable that referees of lesser mainstream journals would succeed where those at a more prestigious journal failed. Therefore, if my paper, and others similar in that they are counter to the prevailing orthodoxy, are to be published, it must be in journals receptive to heterodoxy. Moreover, I would not even consider submitting the instant paper to a mainstream journal of any rank. I cannot imagine such an attack against a top-tier mainline journal ever being published in such a journal. And yet, if in fact referees and editors at top-tier mainstream journals are incompetent in any relevant area this is important for the profession to know.

I conclude, therefore, that there is a need for specifically Austrian journals, not only because of the bias of mainstream journals, but also because Austrians should not have to subject their work to referees and editors incompetent in the very area of their supposed expertise.

Appendix

Referee #1 took the equation, $U = 699 - 216 \ln(r)$, from an example in Spiegel (1967, pp. 97–98.) (Please note that I used a different edition, Spiegel (1981, pp. 103–04).) The example is formulated as a problem with three parts. The "[s]olution . . . $U = 699 - 216 \ln(r)$," which the referee took from the book, is but the solution to one part thereof. I have reproduced the relevant portion of the example immediately below. (Emphasis in original.) Subsequently, I restate, in expanded form, the example in a way that explicates the referee's error.

> The amount of heat per unit time flowing across an area A is given by
>
> $$q = -KAdU/dn \qquad (3)$$
>
> The constant of proportionality K, used above, depends on the material used and is called the *thermal conductivity*. The quantity of heat is expressed in *calories* in the cgs system, and in *British thermal units*, *Btu* in the fps system. [Because of the confusion that arose from the use of "pound" as a unit of mass and as a unit of force, in the modern version of the fps (foot-pound-second) system, the BE (British Engineering) system, the slug, not the pound, is the unit of mass.] Consider now an illustration using the above principles.
>
> A long steel pipe, of thermal conductivity $K = 0.15$ cgs units, has an inner radius of 10 cm and an outer radius of 20 cm. The inner surface is kept at 200° C and the outer surface is kept at 50° C. (a) Find the temperature as a function of distance r from the common axis of the concentric cylinders.

[23]In fact, when I submitted the paper for review a colleague and I made a wager as to how quickly it would be rejected. As I recall, I said something like eight weeks or less and my colleague said more than that; he won by about two days.

(b) Find the temperature when $r = 15$ cm. (c) How much heat is lost per minute in a portion of the pipe which is 20 m long?

MATHEMATICAL FORMULATION (Spiegel 1981, pp. 103-04). It is clear that the isothermal surfaces are cylinders concentric with the given ones. The area of such a surface having radius r and length *1* is $2\pi rl$. The distance dn is dr in this case. Thus, equation (3) can be written

$$q = -K(2\pi rl)dU/dr \quad (4)$$

Since $K = 0.15$, $l = 20$ m = 2000 cm, we have

$$q = -600\pi r\, dU/dr \quad (5)$$

In this equation, q is of course a constant. The conditions are

$$U = 200° \text{ C at } r = 10,\ U = 50° \text{ C at } r = 20 \quad (6)$$

SOLUTION. Separating the variables in (5) and integrating yields

$$-600\pi U = q \ln r + c \quad (7)$$

Using the conditions (6), we have $- 600\pi(200) = q \ln 10 + c$, $- 600\pi(50) = q \ln 20 + c$ from which we obtain $q = 4O8,000$, $c = -1,317,000$. Hence, from (7) we find

$$U = 699 - 216 \ln r. \quad (8)$$

Expanded Restatement. The foregoing material is restated with the units, in brackets, explicitly attached to the algebraic symbols for the variables. The dimensions of thermal conductivity are units of: energy/(time • distance • thermodynamic temperature). Therefore, K, the thermal conductivity of the pipe, is, in (cgs) units, cal/(sec • cm • °C). The units of the other relevant variables are: cm for l, dr, and r; °C for dU and U; and, cal/sec for q.

Rewriting equation (4) yields:

$$q[\text{cal/sec}] = -K[\text{cal/(sec}\cdot\text{cm}\cdot°\text{C)}]\cdot(2\pi r[\text{cm}]l[\text{cm}])\cdot dU[°\text{C}]/dr[\text{cm}] \quad (4')$$

Substituting the values for K and l yields:

$$q[\text{cal/sec}] = -0.15[\text{cal/(sec}\cdot\text{cm}\cdot°\text{C)}]\cdot 2\pi r[\text{cm}]2000[\text{cm}]\cdot dU[°\text{C}]/dr[\text{cm}] \quad (5')$$

The cm unit and the °C unit in the denominator of the dimension of K cancel the cm unit in the numerator of the dimension of l and the °C unit in the numerator of the dimension of dU, respectively. Note particularly that because the units of r and dr are identical, cm, and because r is in the numerator and dr in the denominator, the units of the variables r and dr cancel out, and all that is left of these variables are their magnitudes; the algebraic symbols of these variables no longer have units attached. Therefore, canceling units yields:

$$q[\text{cal/sec}] = -600[\text{cal/(sec)}] \cdot \pi r \cdot dU/dr \qquad (5'')$$

At this point, the only units that have not canceled out are cal/sec on both sides of the equation.

Rewriting (5″) to put it in integrable form yields:

$$-600[\text{cal/(sec)}]\pi dU = q(\text{cal/sec})\ dr/r. \qquad (5''')$$

The integration of dr/r yields ln r, but the algebraic symbol for the variable r in the term ln r has been shorn of its units and only its magnitude remains, and, therefore there are no units to be operated on by the ln operator. Similarly, the integration of dU yields U, but the algebraic symbol for the variable U has been shorn of its units and only its magnitude remains. The solution to (5‴), then, is:

$$-600[\text{cal/(sec)}]\pi \text{U} = q[\text{cal/sec}] \ln r + \text{c} \qquad (7')$$

Recall the conditions (6) i.e., U = 200° C at r = 10 [cm] and U = 50° C at r = 20 [cm], while remembering that r refers only to the relevant magnitudes at this point; the cm appear in brackets only as a reminder of the dimensions that r had prior to their being canceled in the equation, $q[\text{cal/sec}] = -K[\text{cal/(sec} \cdot \text{cm} \cdot {}^\circ\text{C)}] \cdot (2\pi r[\text{cm}] l[\text{cm}]) \cdot dU[{}^\circ\text{C}]/dr[\text{cm}]$. Then, substituting these conditions into (7′), and solving for q and c yields: q = 408,000 cal/sec and c = –1,317,000 cal/sec. Note: the units of c are, necessarily, cal/sec, else dimensional analysis would yield inconsistent units, an absolutely certain sign of error, to wit: an incorrect relation.

In order to solve for U including, the appropriate units, rewrite (7′) (substituting q = 408,000 cal/sec for q and c = –1,317,000 cal/sec for c) as:

$$-600[\text{cal/(sec} \cdot {}^\circ\text{C)}]\pi\ U[{}^\circ\text{C}] = 408{,}000\ [\text{cal/sec}] \ln r - 1{,}317{,}000\ [\text{cal/sec}] \qquad (8')$$

Isolating the U term yields:

$$U[{}^\circ\text{C}] = (408{,}000\ [\text{cal/sec}] \ln r - 1{,}317{,}000\ [\text{cal/sec}])/(-600[\text{cal/(sec} \cdot {}^\circ\text{C)}] \cdot \pi) \ (8'')$$

As the units cal/sec appear in every term in the numerator and in the denominator of the right-hand side of 8″, they may be canceled, yielding:

$$U[{}^\circ\text{C}] = (408{,}000\ [{}^\circ\text{C}] \ln r - 1{,}317{,}000\ [{}^\circ\text{C}])/(-600\pi) \qquad (8''')$$

or

$$U[{}^\circ\text{C}] = 699[{}^\circ\text{C}] - 216[{}^\circ\text{C}] \ln r. \qquad (8'''')$$

Because, as previously shown, ln r is dimensionless, it is obvious that the dimensions on both sides of equation (8⁗) are degrees Celsius, and, therefore, identical, as must be the case.

However, equation (8′′′′) is the very solution (*cum* appropriate units) that the referee cited in his report, to support his position.

Therefore, both Referee 1's undeniable implication that the units on the different sides of the equation, $U = 699 - 216 \ln r$, are not the same, in that the units on the left-hand side are degrees Celsius and the units on the right-hand side are centimeters, and his conclusion "that physics contains the same 'defect' [i.e., the failure to properly account for units and, therefore, the inconsistency between the two sides of equations] when certain systems are examined, are incorrect. *QED*.

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