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ARTICLES

Mathematics, Metaphors and Economic Visualisability

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In the nineteenth century, scholars trained in both mathematics and political economy began to construct economic theory in mathematical form. Over the course of the next half century, mathematical analysis gradually came to be applied, sometimes implicitly and often relegated to appendices and footnotes, in more and more parts of economic theory. Then, quite suddenly, in the years immediately following the Second World War, the mathematization of economic theory gathered pace. So forceful and all-encompassing was the subsequent transformation in the way in which economic theory was constructed that a mere three decades later it would be difficult to find a branch of economic theory untouched by mathematical analysis.

Despite the pervasiveness of mathematics in economic theory, many important methodological issues relevant to the deployment of mathematics in economics remain untouched in the economics literature. Part of the reason for this is that debate over the utilization of mathematics in economics has been narrowly focused on the appropriateness of the deductive method—the proof of many theorems that logically flow from a few axioms. As a consequence of this narrow focus, the nature and role of mathematics in the development of economic science and knowledge has not received the attention that it deserves. Important fundamental questions have gone unanswered. What does mathematical economics mean? What is its nature? Can it help solve any economic problem? Does it do more harm than good? Despite Samuelson's (1952) declaration that "Mathematics *is* language," there have been only a few attempts to explore such questions using the tools of linguistic theory and cognitive psychology.

In this paper, it is shown how linguistic theory and the closely related investigations of cognitive psychologists illuminate the path that must be taken to generate answers to the questions that the utilization of mathematics in economics has posed for economists. Mathematical models in economic theory, like all models

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in science, involve the construction of metaphor. The models are not literally true. Mathematical economics creates analogies between economic elements and mathematical objects that compel economists to think about their principal subject (economic reality) in terms of a subsidiary subject (mathematics). In this way mathematical economics may generate new insights about the nature and operation of the economic system or identify previously undiscovered connections between elements of economic reality. Importantly, the metaphorical nature of mathematical economics means that mathematical economics must be judged, not just on the rigor of its proofs, but on the appropriateness of the analogies it creates between mathematical objects and elements of economic reality.

There are no set criteria for judging the appropriateness of mathematical metaphors. However, an especially important property of metaphor is its ability to invoke images in the minds of readers or listeners. These images may be either clear diagrammatic pictures or they may be images of a more kinaesthetic or intuitive nature. In this paper, the image-invoking property of metaphor is called *visualisability.* Good metaphors in everyday language and in science are vivid metaphors. Vivid metaphors provide an effective way of communicating new scientific discoveries. More importantly, when it comes to the particular case of mathematical economic theory, visualisability means vivid connections between mathematical objects and economic elements. Visualisability is an important property of metaphor that demands the consideration of economists. This is especially the case given the increasing utilization of abstract mathematics in modern economic theory. With increasing abstraction, the connections between mathematical objects and economic elements may become clouded and stretched and this might potentially undermine the effectiveness of mathematical metaphors in economics.

This paper is organized as follows. In "Metaphor", a definition of metaphor is presented and the necessary linguistic theory is introduced. In "Mathematical Economics and Metaphor", the metaphorical nature of mathematical economics is discussed. A delimitation of metaphor is applied to mathematical economics to provide support for the claim that mathematical economic theory involves metaphor construction. In "The Nonequivalence of Mathematics and Literal Prose in Economics", the argument that mathematical economics and literary economics are strictly equivalent is examined. Although this argument is raised in many discussions about mathematics in economics, it is wrong. Mathematical and literary economics are not strictly equivalent. Mathematical economics has a non-literal, metaphorical quality that does not have a strict equivalent within the realms of plain (literal) prose. In "Metaphor and Advances in Understanding", the ways in which metaphor can help to advance understanding are discussed. In "Metaphors as Retardants of Advances in Understanding", the ways in which metaphor may retard understanding are discussed. In "Economic Visualisability", the importance of economic visualisability is identified. The paper is concluded with "Conclusions".

Metaphor

Metaphor is one category of non-literal or figurative language. In a metaphor, an object or event is described using concepts that cannot be applied to that object or event in a conventional way (Indurkhya 1992). The object or event that is described is called the *principal subject* and the concept that cannot be applied to that object or event in a conventional way is called the *subsidiary subject*. Some examples of metaphor are provided below:

(a) My lawyer is a *shark*
(b) Rising unemployment in this city is a *time bomb*
(c) The king is a *lion*
(d) The *death* of literature; and, from Shakespeare
(e) Juliet is the *sun*; and
(f) All the world's a *stage*
And all the men and women merely *players*

In written or spoken language, metaphors generally introduce a marked contrast between the words used metaphorically in a sentence and the words used literally. For example, in the metaphorical expression, "Rising unemployment in this city is a time bomb," there is a distinct contrast between "time bomb" and the remaining words in the sentence. The contrast arises because "time bomb" cannot be applied to the phenomenon of rising unemployment in a meaningful way whilst retaining its literal meaning (as an explosive device triggered by a timing mechanism).

Although the pervasive utilization of metaphor in human language raises many important questions, the dominant theme in the literature is *meaning*. Reading through the list of metaphorical expressions (above), one can understand what is meant by them. These metaphorical expressions have meaning. The important problem for scholars is to provide an explanation for that meaning. In the exegesis of metaphor, two predominant categories of theory have emerged: (1) monistic theory; and (2) dualistic theory. Monistic theorists contend that words used metaphorically do not keep their antecedent literal meaning and refer only to their new non-literal context. Dualistic theorists, on the other hand, contend that words used metaphorically maintain a dual reference to both their antecedent literal context and a new non-literal context (Mooij 1976). Dualistic theories have emerged as those most likely to provide solutions to the problems that metaphor poses for linguistic theorists, philosophers, psychologists and methodologists.

The most straightforward dualistic theory is the substitution view. According to the substitution view, metaphors are simply substitutes for literal expressions. For example, the speaker who uses the metaphorical expression, "The death of literature," could, according to the substitution view, have used a literal expression that means the same thing. The word "death" in this case communicates something that could have been expressed literally. The metaphor is simply used in a novel or poetic way to decorate and embellish one's speech. The substitution view implies that metaphorical expression performs no greater function than to please and amuse readers or listeners. Once scholars came to realize, however, that metaphors may perform a more important and complex role in the communication of ideas it was necessary to develop an alternative theory of metaphor. We find one such attempt in what has been called a special case of the substitution theory: the comparison theory.

The comparison view is one of the most popular dualistic theories of metaphor. According to comparison theory, metaphor is abridged or implicit comparison between principal subject and subsidiary subject (Mooij 1976). A proponent of the comparison view holds that metaphor is the presentation of an analogy or similarity (Black 1955). Consider the meaning of the metaphor, “My lawyer is a shark.” According to a comparison theorist, the speaker who says that his lawyer is a shark wants to tell his listener something about the lawyer. Rather than formally compare his lawyer to an aggressive, vicious, brutal, tenacious, and fierce creature, the speaker presents the analogy in the form of this non-literal or figurative expression (Glucksberg et al. 2001). The listener, if he detects the analogy between the lawyer and the shark, may determine the speaker's intended (literal) meaning (Black 1955).

The comparison theorists recognize the importance of analogy in metaphor and, as mentioned previously, view metaphor as the presentation of a similarity or analogy. At first this appears to be quite an attractive theory but there is a problem. If a metaphor states that “A is B,” there must, according to the comparison view, be some a priori literal resemblance between the principal subject A and the subsidiary subject B. The articulation of the resemblance, analogy or similarity in a metaphorical expression could, if the comparison view is correct, be held to be a surrogate for a formal comparison (Black 1955). This is troublesome for the following reason. There is often no a priori literal resemblance between A and B. Before the metaphor is constructed there is no possibility for formal comparison of A with B. It is the metaphor that *creates* the similarity or analogy (Black 1955). This leads us to the interaction theory of metaphor.

In the interaction view of metaphor, rather than being seen as comparisons based upon a pre-existing analogy or similarity, metaphors are supposed to create or induce analogies between principal subject and subsidiary subject. The principal subject and subsidiary subject are regarded as systems rather than isolated words (Indurkhya 1992). For example, in the metaphorical expression, “The king is a lion,” it is not just the word “lion” but all our knowledge and beliefs about lions that act on the principal subject (the king). This knowledge and these beliefs are the systems of the two subjects (Indurkhya 1992). Upon hearing or reading a metaphorical expression, the system of the subsidiary subject—all of the reader or listener's knowledge and beliefs about the subsidiary subject—interacts with the system of the principal subject—all of the reader or listener's knowledge and beliefs about the principal subject—in the listener or reader's thoughts. The result of this interaction is that the principal subject looks more similar to the subsidiary subject. A similarity or analogy is created.

The interaction view implies that there is no simple basis for metaphor. Metaphor is not simply the presentation of a pre-existing analogy or similarity between principal subject and subsidiary subject but has a more important role in the communication of ideas and is possessed of a more complex cognitive function. Words utilized metaphorically fulfil a cognitive role that is not equivalent to the function fulfilled by possible literal surrogates. In metaphor, the systems of the principal subject and subsidiary subject work together or interact. As a result, attributes of the subsidiary subject are acquired by the principal subject and the reader or listener comes to understand the principal subject in a new and different way. Metaphor was once viewed as a purely literary phenomenon. Whilst the interaction view of metaphor must be complemented by the experimental and theoretical investigations of cognitive psychologists to provide a complete exposition of metaphor meaning and comprehension, the interaction view has made it clear that metaphor is not simply a part of the way in which people speak but also a part of the way in which people think.

Mathematical Economics and Metaphor

The role of metaphor in economics has been recognized by several distinguished contributors to the literature, including McCloskey (1983, pp. 502–07), Mirowski (1994) and Klamer and Leonard (1994). The discussions presented by McCloskey and the other authors draw upon some of the same linguistic theory that was presented in the previous section of this paper. In particular, the interaction theory acquires a place of prominence not only because of its usefulness in explaining how metaphors work but also because of its bringing to the forefront of the analysis the "cognitive" role of metaphor. McCloskey (1983, p. 503) and Klamer and Leonard (1994, pp. 27–30) recognize that metaphors are not "ornamental" but play an important and useful cognitive role. In particular, the role that metaphor performs as the vehicle of "mutually advantageous" illumination of ideas from different contexts is highlighted. This paper complements these existing contributions to the literature on metaphor and economics by exploring slightly more fully, and more formally, the particular case of mathematical economics as metaphor.

A basic interaction view of mathematical economics as metaphor may proceed as follows. Consider the statement, "The list of commodity prices is a vector in commodity space." A person who speaks of a vector is referring to a mathematical object with certain properties. Likewise, a person who speaks of a list of commodity prices is referring to an economic element with certain properties. As described above, the metaphor invokes two systems: (1) all of the listener's knowledge and beliefs about vectors; and (2) all of the listener's knowledge and beliefs about the list of commodity prices. These two systems interact in the listener's thoughts. Providing that the metaphor is appropriate, the suitable listener will be able to make the assertions implied in the metaphor fit the principal subject (Black 1955). The result is that the list of commodity prices in the economic system looks more like a vector.

While the interaction view sheds some light on the operation of mathematical metaphors in economics, it does not explain on its own how the mathematical metaphors are processed or comprehended. Black (1979a, p. 192) explains,

> The chief weakness of the "interaction theory," which I still regard as better than its alternatives, is lack of clarification of what it means to say that in a metaphor one thing is thought of (or viewed) as another thing....To think of God as love and to take the further step of identifying the two is emphatically to do something more than to compare them as merely being alike in certain respects. But what that "something more" is remains tantalizingly elusive: we lack an adequate account of metaphorical thought.

The investigation of metaphor processing and comprehension has been an ongoing research program for much of the past 25 years. It has attracted the attention of literary theorists, cognitive psychologists, and psycho-linguistic experimentalists. The result has been the development and experimental analysis of a number of different models of metaphor processing and comprehension.

The *standard pragmatic model*, which was among the earliest models developed, proposes that metaphor comprehension takes place in distinct phases in which (1) the listener determines the literal meaning of the statement; (2) the listener decides if the literal meaning is the intended meaning of the statement; and (3) if the listener decides that the literal meaning is inappropriate he or she must determine the metaphorical meaning (Gibbs and Gerrig 1989, p. 146). However, the results of experiments to test the validity of the standard pragmatic model have generally been inconsistent with the model (Ortony et al. 1978). That is, the participants in these experiments did not take longer to comprehend a metaphorical statement than a literal statement. If metaphor comprehension follows an attempt to first determine the literal meaning, metaphors should take longer to comprehend than literal statements (Gibbs and Gerrig 1989, p. 147). This has been found *not* to be the case.

While it may now be possible to conclude that metaphor comprehension does not occur in distinct phases, it remains to be seen whether the same or different mental processes facilitate the comprehension of metaphorical expressions and literal expressions. We still need an answer for the question concerning what is happening in the mind of the listener as he or she attempts to comprehend a metaphorical expression. A number of models have been developed. These include category-based models (Keysar and Glucksberg 1992; Thomas and Mareschal 2001), comparison-based models (Chiappe and Kennedy 2001; Gagne 2002), mapping models (Gentner and Bowdle 2001), and conceptual models (Lakoff and Johnson 1980; Gibbs 1992; Coulson and Matlock 2001). While research is ongoing, most of the challenges faced in the development of metaphor processing models appear to be mitigated by the deployment of conceptual models (Ritchie 2003). In conceptual models, "cross-domain mappings" are accorded a place of prominence.

The conceptual metaphor theory (CMT) developed by Lakoff and Johnson (1980) suggests that metaphors are the output of a cognitive process that involves the systematic—rather than ad hoc—utilization of pre-existing cross-domain mappings in which we understand the principal domain by exploiting an analogically related subject, the subsidiary domain (Coulson and Matlock 2001, p. 297). CMT explains why, for example, groups of metaphorical expressions have emerged that all (systematically) explain life (the principal domain) in terms of a journey (subsidiary domain): "He's at the cross-roads," "It's been a long, hard road," "Look how far she's come" (Gibbs 1992, p. 573). According to Coulson and Matlock (2001, p. 297), "The systematicity in the use of [subsidiary] and [principal] domain terminology derives from the fact that some of the logic of the [subsidiary] domain has been imported into the [principal domain] in a way that maintains the mappings from one to the other." Metaphors are processed and comprehended by inviting the systematic deployment of pre-existing cross-domain mappings. This deployment is the interaction referred to imprecisely in the interaction view of metaphor.

Let us apply CMT to mathematical economics. Consider the standard individual utility function deployed in microeconomic theory:

$$\text{utility} = U(X_1, X_2, \ldots, X_n) \tag{1}$$

According to CMT, the metaphorical re-description of individual satisfaction in the context of choice among combinations of Xs invites the listener or reader to utilize pre-existing cross-domain mappings such that he or she understands the principal domain in terms of the analogically related subsidiary domain. CMT explains why, for example, functions are systematically deployed in mathematical economics. This is because some of the logic of functions has been imported into the principal domain (economics) such that mappings can be maintained from one metaphorical re-description of an economic element as a function to the next. Hence, "[Mathematical economics] is the manifestation of conceptual structure organized by a cross-domain mapping—a systematic set of correspondences between the source and target that result when cognitive models from a particular [subsidiary] domain are used to conceptualize a given [principal] domain" (Coulson and Matlock 2001, p. 297). This is how mathematical metaphors in economics work.

Certain questions arise at this point. "How do we recognize a metaphor?" and "How can we be sure that mathematics in economics is metaphorical in nature?" The identification of metaphor is not an easy task. While some scholars believe that a diagnostic tool might be developed to fulfil this duty, others have suggested that the identification of metaphor relies to a large extent on our knowledge of what is to be a metaphorical statement and an assessment of whether a metaphorical reading is preferred to a literal one (Black 1979b). But however imperfect a set of diagnostic criteria might be, the metaphorical nature of mathematical economics may still be appreciated more fully through the application of a delimitation of metaphor.

One such delimitation of metaphor is extant in the literature. This is Mooij's (1976) set of criteria that may be deployed to identify metaphor. According to Mooij (1976, p. 26), a statement can be classed as metaphorical if:

(a) The context in which the statement is made makes it clear that the statement is substantially about a certain principal subject, *A*.
(b) The words, *W*, whose metaphoricalness is under discussion, have a field of literal descriptive meaning, *F*.
(c) These words *W* are used in the statement in such a way that at least part of their function seems to be a direct description, characterization, or indication of certain aspects of *A*.
(d) Although *A* and *F* may only be vaguely circumscribed, it has to be clear that the aspects of *A* meant in (c) do not show the features *F*.
(e) The statement is not to be interpreted as simply false, inappropriate or nonsensical (which it would be on the basis of a literal reading of *W*), because it is understandable as a contribution to the discourse about *A*. The metaphorical words do not only seem to give information about *A*, according to (c), but they actually help to do so.

To Mooij's (1976) schema may be added an additional criterion. This additional criterion reflects what has been said in the previous section on the theory of metaphor, particularly the interaction view.

(f) Upon reading or listening to the statement an analogy or similarity between the principal subject and subsidiary subject is illuminated or becomes apparent or, to be more precise, we are invited to systematically utilize a pre-existing cross-domain mapping.

While this set of criteria might be applied to the written and spoken statements that one comes across in everyday life, it need not do so exclusively. This delimitation of metaphor may be applied just as easily to the sentences, words, and equations of mathematical economic theory. Mooij (1976, p. 26) confirms that this delimitation may apply whenever F is not realized within A. In mathematical economics, the mathematics has a field of literal (mathematical) meaning that is certainly not realized within the principal subject (economic elements). Vectors, for example, have a field of literal mathematical meaning as elements of a vector space. This field of literal mathematical meaning is not realized in the economic elements that vectors are sometimes used to explain.

To apply Mooij's (1976) schema, some examples are required from mathematical economics. The papers of Debreu (1984, 1991) on the mathematization of economic theory provide a ready source of examples. In his summary of the mathematization of economic theory, Gerard Debreu explained how the problem of determining the existence of general equilibrium was approached by him and others. Significantly, the development of a successful solution to the problem relies on the creation of an analogy between the list of commodities in the economic system and a mathematical object called a vector space. Debreu (1984, p. 268) explained:

> One makes a list of all the commodities in the economy. Let l be their finite number. Having chosen a unit of measurement for each one of them, a sign convention to distinguish inputs from outputs (for a consumer inputs are positive, outputs negative; for a producer inputs are negative, outputs positive), one can describe the action of an economic agent by a vector in the commodity space R^l. The fact that the commodity space has the structure of a real vector space is a basic reason for the success of the mathematisation of economic theory. In particular convexity properties of sets in R^l, a recurring theme in the theory of general economic equilibrium, can be fully exploited. If, in addition, one chooses a unit of account, and if one specifies the price of each one of the l commodities, one defines a price-vector in R^l, a concept dual to that of a commodity-vector. The value of the commodity-vector z relative to the price-vector p is then the inner product $p \cdot z$.

In this passage, Debreu touches on most of the main components of the general equilibrium analysis he and others helped to develop. Consider the following statements: (1) the environment in which agents undertake their economic actions is a commodity (vector) space; (2) the action of an economic agent is a vector in commodity space; (3) the prices of commodities are a vector in commodity space; and (4) the commodities in the economy are a vector in commodity space. These look very much like metaphors. Certainly they appear to create analogies between specific elements of economic reality and mathematical objects (vectors and vector spaces). Let us examine this in more detail.

Consider the specific statement, "The environment in which agents undertake their economic actions is a commodity vector space." It is appropriate to choose this particular statement because it is really the critical analogy upon which much mathematical economic theory has been constructed. The application of our delimitation of metaphor to this statement yields the following insights:

(a) The context (especially the theorists' intentions) makes it clear that the statement (the theory) is about a certain principal subject. The theorists' intentions are certainly to tell us something about the environment in which economic agents operate. The theorists' intentions are *not* to say something about a mathematical object. The vector space is the subsidiary subject;
(b) The words (or equations) whose metaphoricalness is under discussion (vector space) do have a field of literal (mathematical) meaning. This field of meaning can be stated as follows. The vector space is a set of vectors and two operations: (1) vector addition; and (2) scalar multiplication;
(c) The words (or equations) are used by mathematical economic theorists in such a way that at least part of their function seems to be a direct description, characterization or indication of certain aspects of the environment in which economic agents operate.
(d) It is clear that the actual (real-world) environment in which economic agents operate does not literally have the features stated in (b). That is, the environment in which economic agents undertake their activities is not literally a set of vectors and two mathematical operations;
(e) The statement that the environment in which economic agents undertake their activities is a vector space is not dismissed as inappropriate or false because it *seems* to be an understandable contribution to the discourse on economic reality; and
(f) Upon reading or hearing the statement, an analogy or similarity becomes apparent (to the suitable reader or listener). If one possesses the necessary knowledge of the subsidiary domain (linear algebra) one can utilize a pre-existing cross-domain mapping and exploit the analogically related subsidiary domain to understand the principal domain.

By creating analogies between economic elements and mathematical objects, the general equilibrium theory developed by Debreu and others changed the way in which economists thought about certain economic elements. Specifically, such economic elements as the environment in which agents operate and the actions of those agents attained certain attributes as a result of thinking about them in terms of the subsidiary subject. In this way, equilibrium theory may be said to involve metaphor construction.

Countless other examples of mathematical metaphors in economic theory may be listed, all of which create analogies between mathematical objects and economic elements. For example, Debreu (1984, 1991) speaks about the perfect fit between the supporting hyper-plane theorem (subsidiary subject) and the problem that concerns the optimal state of an economy (principal subject) as well as the possibility of casting welfare economics (principal subject) in set-theoretical terms (subsidiary subject). Demand and supply functions, production functions, utility functions, indifference curves, and budget constraints. All of these pieces of mathematical economics, along with the others discussed in this section, create analogies between mathematical objects and economic elements (and invite the utilization of the cross-domain mappings we spoke of earlier). In so doing, they compel economists to see or understand these economic elements in a new and different way. Mathematical economics is a collection of metaphors that create analogies between various mathematical objects and economic elements.

The Nonequivalence of Mathematics and Literal Prose in Economics

The metaphorical nature of mathematics in economics has many implications. One of these implications concerns the equivalence of mathematical symbols and literary prose in economics. An argument that is commonly found in discussions on mathematics in economics and in discussion defending mathematical economics concerns the equivalence of mathematical symbols and literary words. Professor Paul A. Samuelson has argued, on at least one occasion, for the strict equivalence of mathematics and prose (Samuelson 1952) and many other mathematical economists have done likewise. Indeed, it is probably true to say that the equivalence of mathematical symbols and literary words has been one of the key arguments deployed in defense of mathematical economics.

The argument that mathematical economics and literary economics are strictly equivalent involves an application of a substitution view of metaphor to mathematical economic theory. The substitution view of metaphor, as explained previously, is a basic theory of metaphor that views metaphorical expressions as substitutes for literal, plain English expressions. Either could be used with the same ultimate effect of communicating an idea. Applied to mathematical economics, the substitution view implies that mathematical economics is a surrogate for plain, literal literary economics. The only advantage is that mathematical economics provides a more efficient or, possibly, a more aesthetically pleasing way of communicating the theorist's ideas.

This view, which is expressed by Samuelson (1952), has also been expressed by Alfred Marshall (1997, p. vi):

> The chief use of pure mathematics in economic questions seems to be in helping a person write down quickly, shortly and exactly, some of his thoughts for his own use: and to make sure that he has enough, and only enough, premises for his conclusions.

Samuelson and Marshall express a view of mathematical economics that is fundamentally equivalent to a substitution view of metaphor: metaphorical expressions (mathematics in economics) are substitutes for literal English expressions that have an advantage of economy of expression. However, in light of subsequent advances in linguistic theory it is clear that the application of a substitution view of metaphor to mathematical economics is not wholly appropriate. A substitution view allows no special role for mathematics in the communication of economic ideas and implies that the mathematics in economics does nothing more than please readers and to provide a short-hand method of conveying economic theory.

Samuelson (1952) has suggested that it is possible to write out in prose all of Whitehead and Russell's *Principia Mathematica*. He is correct. The reason why this is possible is that in the case of pure mathematics (or logic) all that is required is a translation or substitution. For example, consider the mathematical function $y=f(x)=3x^2$. Translated into literary words, $y=f(x)=3x^2$ means that, given a value for x, we would follow the rule of squaring that value and multiplying by three in order to obtain the corresponding value for y. This is *literally* what this function means. Although the task will become more arduous with more complicated mathematics, it will always be possible to undertake this translation or substitution process. In pure mathematics there is only one subject. A pure mathematician creates no analogies between mathematical objects and elements observed in nature. With regards to mathematical economic theory, however, literal translation of mathematics into words is no longer possible. A mathematical economist deals with two subjects simultaneously and creates analogies between them. In the case of mathematical economics, it is not translation that is required but *explication*. And the explication of metaphors is no easy task.

In plain, literal prose, what does the mathematical demand function $Q_d=a-bP$ mean? This is a far more difficult question than it first appears. Instinctively, one begins to search for an economic meaning hidden within the mathematics waiting to be deciphered. However, one soon realizes that this is a blind alley and if we continue trying to formulate an answer to the question until a satisfactory conclusion is reached, we shall eventually discover that what we have arrived at is not a literal translation of $Q_d=a-bP$ but an *explication* of what about the nature of economic reality the metaphor evokes or brings to our attention. This occurs because there is not a hidden economic meaning within $Q_d=a-bP$ that can be expressed in plain, literal prose. The cognitive-linguistic role that this mathematical metaphor fulfils is more complex than this. It makes us see elements of economic reality in a new and different way. The prices at which economic agents will demand certain quantities become more like a mathematical object called a function. This is something that plain, literal prose does not do (Davidson 1979, p. 43).

Mathematical metaphors in economics do not have a hidden meaning that, once deciphered, can be expressed in plain, literal prose. Mathematical metaphors in economics, like other types of metaphor, have an *effect* on the reader or listener (Davidson 1979, p. 43). An explication of mathematical metaphors in economics would therefore involve helping a less suitable reader or listener see what the mathematical economic theorist wants his or her suitable readers or listeners to see. Such cannot be accomplished by simply translating the mathematics into words or, equivalently, substituting plain, literal prose for the non-literal (mathematical) statements. In mathematical economics, mathematical symbols do not simply stand in as surrogates for literary words. Mathematical economics creates analogies between economic elements and mathematical objects. *In economics* there is *not* a strict equivalence between mathematics and words. The metaphorical nature of mathematical economic theory gives it a special function in the communication of ideas. It compels economists to see and think about economic elements in a new and different way.

The special cognitive-linguistic function of metaphor in language is present in metaphorical mathematical economics. This cognitive-linguistic function is a powerful tool of language that can bring forth new insights or improved understanding. To suitable listeners these new insights may appear especially beautiful and attractive. With regards to mathematical economic theory, these suitable listeners will usually be other mathematical economists. Here lies at least part of the reason for the mathematization of economic theory that has been witnessed over the past century or so. To suitable listeners, mathematical economics, deploying the strength of metaphor as a tool in the development of scientific understanding, has generated striking and beautiful results. The logic of mathematics has been imported into the principal domain (economics) such that mappings can be maintained from one mathematical re-description of an economic element to the next. The systematic exploitation of this organized conceptual structure holds out the promise of more, equally beautiful results. This has contributed to the considerable allure of mathematics to some theorists and has been responsible for the acceptance of mathematical results as advancements in our knowledge about the economic system.

Metaphor and Advances in Understanding

The view had long been held that metaphors have no place in science or even in serious discourse. There are many well-known philosopher opponents of metaphor, including Hobbes, Nietzsche, Mauthner, Flaubert, and Robbe-Grillet (Mooij 1976). Most notable, perhaps, is Locke whose disdain of metaphor is reflected in the passage reproduced below:

> Since wit and fancy finds easier entertainment in the world, than dry truth and real knowledge, figurative speeches and allusions in language will hardly be admitted as an imperfection or abuse of it. I confess, in discourses where we seek rather pleasure and delight than information and improvement, such ornaments as are borrowed from them can scarce pass for faults. But yet, if we would speak of things as they are, we must allow that all the art of rhetorick, besides order and clearness, all the artificial and figurative application of words eloquence hath invented, are for nothing else but to insinuate wrong ideas, move the passions, and thereby mislead the judgement, and so indeed are perfect cheats. (Locke 1824, book 3, ch. 10, Section 34).

Even Aristotle, who thought "by far the most important matter is to have skill in the use of metaphor," (*Poetics* 1459*a*) expressed caution about the harm to understanding that might be caused by its misuse. However, the view that metaphors have no role in the development of science and knowledge is changing, though in economics the issue has not really been widely discussed.

In science in general, metaphors might fulfil a number of important functions. At a most fundamental level metaphors provide a vehicle for the communication of new scientific theories (Boyd 1979). However, it is also widely believed that metaphors are actually essential to the construction of theories. Indeed, according to some, scientific models are metaphors (Hesse 1966; Bradie 1998; Kittay 1987; Kuhn 1979). The models in scientific theories are not literally true. Rather they are devices that organize our experiences and our current state of knowledge. The theoretical model is the subsidiary subject through which elements of the empirical domain, the principal subject, are explained. In the process of model construction and application, the empirical domain obtains a set of new attributions as a result of scientists' thinking about it in terms of the theoretical model. Elements of the empirical domain are redescribed in terms of the model (Hesse 1966; Bradie 1998).

Hesse (1966) fully explores these ideas. She argues that scientific models as metaphors play a critical role in redescribing and explaining the principal subject and highlights the importance of metaphor as a means for overcoming the absence of appropriate language with which to discuss new scientific discoveries. Hesse (1966) also identifies a particular role of analogy in scientific development. As mentioned throughout this paper, metaphors may create analogies between a principal subject and subsidiary subject. Hesse (1966, pp. 101–29) describes how these similarities, once identified, might justify the prediction of further analogies between the two subjects (Indurkhya 1992). The arguments that may be derived through this process of analogical prediction cannot be deduced from existing knowledge (Indurkhya 1992, p. 316). That is, the conclusions that may be drawn are "*new* knowledge beyond the logical limits of existing knowledge" (Indurkhya 1992, p. 316).

An example of scientific model as metaphor is to be found in the deployment of the Brownian motion model in financial economics. The Brownian motion model developed by physicists has been used to explain the behavior of asset prices on the stock exchange. Here, a theoretical (Brownian motion) model of asset prices is the subsidiary subject through which the principal subject (empirical asset prices) is explained. In so doing theorists redescribed the behavior of stock market prices using the previously developed Brownian motion model from physics and in the process created an analogy between asset prices and the motion of microscopic particles. In one's thoughts, the systems of the two subjects—all of one's knowledge and beliefs about the motion of particles on the one hand and the movement of asset prices on the other—interact. Asset prices are now seen differently. Asset prices acquire the set of attributes of Brownian motion. They are viewed as independent and identically distributed events.

Something may be said here of the improvements that (metaphorical) economic and scientific models may undergo over time. The Brownian motion model of asset prices is, in its basic characterization, a very poor metaphor. The Wiener process—the mathematical idealization of Brownian motion—exhibits continuity but not smoothness. That is, there are no breaks or gaps in the path that a particle traverses but the path is very jerky and haphazard. Asset prices also exhibit jerky and seemingly haphazard movements. However, asset prices are not always continuous and can (and do) experience gaps from one price to another price without "touching" any prices in between. This tendency of asset prices to "jump" has, however, been accounted for in the more sophisticated metaphorical redescriptions of asset prices as Wiener Brownian motion processes developed by modern finance theorists. It is certainly the case that metaphorical redescriptions are far from static phenomena and must be judged carefully with potential for improvement acknowledged.

Metaphor has proven so useful because it enables us to think about one thing in terms of another. That is, it is a cognitive apparatus. The creation of analogies between a principal subject in which increased understanding is sought and a subsidiary subject may contribute new information or insights about the principal subject. Metaphor is a device that may be usefully deployed whenever new objects or situations are being investigated. This utility derives from the ability that metaphor gives scientists and other scholars to explain the principal subject, which may be a new field of research, in terms of a subsidiary subject that has already been thoroughly investigated and explained. We need categories and concepts to structure our thoughts. When the problem is new, characterizing it in terms of something familiar may greatly assist the development of our understanding (Pylyshyn 1979).

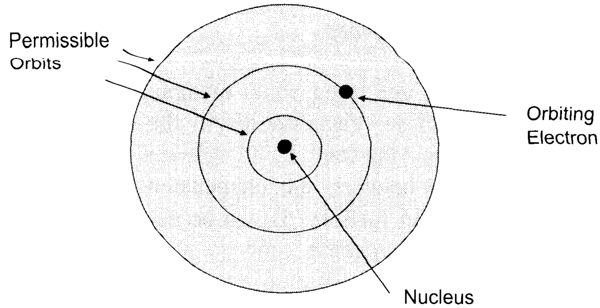
In addition, metaphors in science serve as heuristic tools providing a procedure for arriving at new solutions and rhetorical devices providing a way to express new or existing ideas in a more effective or persuasive manner (Bradie 1998). When the development of knowledge in a particular area stalls because the presently deployed tools of investigation have been exhausted, the creation of analogies between this principal subject and a subsidiary subject may permit the generation of new solutions. Scholars operating at the frontiers of their fields may deploy the rhetorical function of metaphor to inform their colleagues of the nature fof their discoveries and persuade their colleagues of the importance of their discoveries. Skill with language and the ability to impress people with such a skill plays no small part in the communication and subsequent acceptance of new ideas. This, then, is how metaphor may assist the development of science and knowledge.

Metaphors as Retardants of Advances in Understanding

On the downside, there are some substantial caveats associated with the use of metaphor of which scientists must beware. Metaphor can create distortions (Bradie 1999). These distortions arise when features of the subsidiary subject are mistakenly attributed to the principal subject. In this case, the cross-domain mappings between principal and subsidiary domains import features from the subsidiary domain into the principal domain that yield a misguided comprehension of the principal subject. For example, in the early twentieth century, physicists were embarking upon an exciting research program that sought to determine the structure of the atom. Niels Bohr, physicist and Nobel Prize winner, proposed a model of the atom that depicts an atom as a small nucleus orbited by electrons in a structure analogous to a solar system, see figure 1:

Unfortunately, the Bohr model, useful and important as it was during its heyday, is an example of an ultimately mistaken attribution of features of a subsidiary subject (solar system) to a principal subject (the atomic structure). The atomic structure is actually rather different from a miniature solar system, a fact that was made clear by the newer generation of quantum mechanics. A second source of distortion derives

Figure 1 The Bohr Model of the Atom



from the tendency of metaphor to emphasize certain characteristics of the principal subject and push others aside. This may be a significant problem if the characteristics that are pushed aside are in fact more important than those that are emphasized.

Omission or neglect of important features of the principal subject is something that must be guarded against. Black (1962) provides a good example. Say that we decide to use the vocabulary of chess to generate a description of war or battle. In so-doing there is the potential for us to neglect certain important aspects of the principal subject (warfare). As Black (1962, p. 42) points out, "The vocabulary of chess has its primary uses in a highly artificial setting, where all expression of feeling is formally excluded: to describe a battle as if it were a game of chess is accordingly to exclude, by the choice of language, all the more emotionally disturbing aspects of warfare." The qualities of the principal subject that are emphasized in metaphor, and those which are not, are important sources of potential problems inherent in the utilisation of metaphor. Mathematical economics is not exempt from these problems.

While mathematical economics may be accepted as an understandable contribution to the discourse on economic reality, by emphasizing those facets of particular economic elements that may be talked about in terms of particular mathematical objects many other possibly more important facets are swept aside. For example, culture is an important component of our world and influences the way in which the economic system functions. Yet it is all but impossible to metaphorically redescribe this particular element of our economic system in terms of mathematical objects.[1] The very choice of mathematical vocabulary by mathematical economists excludes important aspects of the world around us from analysis. Just as metaphor is only one component of language that can be utilized to express ideas, mathematical economics is only one component of language that can be utilized to convey meaningful insights into economic reality. However, the meaning of mathematical economics will not be clear if the mathematical economic theory lacks *visualisability*.

[1] Even if it were possible to some extent, only those facets of culture that could be talked about in mathematical terms would be emphasized.

Economic Visualisability

Visualisability is one thing about metaphors that we must discuss before drawing this paper to a close. Visualisability is the ability of metaphors to invoke images.[2] Visualisability is important in all sciences that deploy mathematical models that create analogies between the elements of a principal subject and mathematical objects. It is useful for scientists to be able to visualize nature in operation. At the beginning of the twentieth century, physicists' biggest challenge as they strove towards a theory of atomic structure was the difficulty in visualising what was happening inside the atom. Bohr's model provided one possible picture of the atomic structure. The physicist confronted with Bohr's model of atomic structure can picture or visualize that structure as a miniature solar system. The Bohr model (metaphor) creates an analogy that permits the visualisation of the principal subject through the filter (Black 1955, p. 545) of the subsidiary subject. The Bohr model provided a starting point for further investigation, both theoretical and experimental.

The images invoked by metaphors need not be clear diagrammatic pictures. They can be more like sensations with a complex kinaesthetic or intuitive property (Gleick 1992). The development of a new theory often begins with such an image invoked in the scientist's own mind as he or she thinks about a principal subject in terms of a subsidiary one. Metaphor is a uniquely powerful way of communicating to others the insight that the scientist has glimpsed and invoking in them the same image of how an element of nature operates. Thinking about metaphor as invoking images leads us back to our discussion on metaphor and meaning. There is not really a hidden meaning within any metaphorical expression but a cognitive content that has no literal equivalent. The created analogy presented to a suitable reader or listener invokes an image. In science, this fulfils the role of connecting elements of nature together and providing new paths to follow toward their explication.

Mathematical models as metaphors have the ability to invoke images. At a fundamental level, mathematical equations may invoke their geometrical counterparts in the thoughts of the reader or listener. In this case, the invoked images may indeed take the form of clean diagrammatic representations. At a more complex level, mathematical equations may invoke images of motion, deceleration, acceleration, and pressure (Gleick 1992). Mathematical equations in physics, for example, may invoke images of the flowing of liquid or, perhaps, images of waves of particles or individual particles moving through space and time or a "field." These invoked images cannot be depicted in a neat diagrammatic picture—indeed the elements of nature that are described by such mathematical models are not literally waves or points in space–time (hence the metaphorical nature of the models). Such images have a kinaesthetic or intuitive property picked up by the "physical intuition" of the physicist.

In economics, visualisability is important. Mathematical economic theory can create analogies between economic elements and mathematical objects that permit economists to see and understand those economic elements in a new and different way. A demand schedule, particularly in its geometric form, is an example of a piece

[2] Aristotle appears to have this quality in mind when he discusses vividness (*Rhetoric*, ch.3. 11).

of mathematical economic theory that has a high level of visualisability. It is very vivid. Upon hearing the statement, "Let the quantities that economic agents will demand of a certain good at certain prices be the function $Q_d = a - bP$," the suitable reader or listener can visualize the quantities demanded by an economic agent at a certain price as an inverse relationship or a downward-sloping curve. When dynamics are added, it is equally easy to visualize demand and supply functions moving fluidly with new equilibria occurring at their intercept. Even in the case of higher dimensions (hyper planes), when there is no recourse to geometry, the equilibrium adjustment process may be sensed kinaesthetically or intuitively as a searching for the variables that simultaneously solve a system of equations.[3]

Economic visualisability is important because it is the images invoked by mathematical metaphors in economics—in the first instance, in the mind of the mathematical economist and in the second instance in the minds of those to whom he communicates his theory—that may help to propel the development of knowledge in our science. Unfortunately, there would appear to be a tendency in contemporary mathematical economics toward low visualisability. As mathematical economics becomes more abstract, the analogies created between economic elements and mathematical objects tend to become stretched and clouded and the distance between the principal subject and subsidiary subject becomes a widening gulf. There is a danger, under such circumstances, that such abstract mathematical economic theory may cease to invoke images that compel economists to see and understand the economic system in a new and different way. In this way, low visualisability or a lack of vividness may diminish the effectiveness of the mathematical metaphors that have proven so important to the development of economic theory.

Perhaps the main contributing factor to low visualisability is the tendency of theorists to create metaphors within metaphors. From the viewpoint of economic theory, once the mathematical analogues for economic elements were created, it became possible to construct, upon this basis, a framework of metaphors within metaphors. As mentioned above, the importation of some of the logic of mathematics into the principal domain of economics facilitates systematic mappings that can be maintained from one (mathematical) metaphorical redescription of an economic element to the next. Debreu (1984, 1991), for example, explains that if all agents in the economy are in equilibrium relative to a given price-vector, the state of the economy is Pareto optimal (Debreu 1984, p. 268). Debreu also speaks about solving the equilibrium existence problem by casting a competitive economy in the form of a game-theoretic system and of how Martin Shubik linked Edgeworth's contract curve to the theory of the core. These are examples of metaphor within metaphor. Edgeworth's contract curve created an analogy and Shubik created an analogy between Edgeworth's metaphor and another mathematical object. The result is that the newly created analogy is now a further step away from the element of economic reality that the theorist really wants to tell us about. Each creation of metaphor within metaphor has the potential to reduce economic visualisability.

[3] The preference for two or three dimensional mathematics that Samuelson (1952) finds difficult to understand, is due to the ease with which such mathematics may be visualized. Having no physical experience with higher dimensions makes mathematical objects like hyper-planes practically unvisualisable for most people.

It might be said that the tendency toward low visualisability is always a distinct possibility whenever mathematics comes to be used in scientific work. The possibility of new knowledge to be discovered through the construction of metaphor within metaphor or the systematic deployment of cross-domain mappings, presents an almost irresistible allure.[4] Mathematical models as metaphors can assist the task of discovering and stating connections between elements of economic reality but it is important to remember that the mathematical metaphors in economics have the potential, like all metaphors, to create distortions or otherwise mislead. Due consideration must be given to these problems as well as to the vividness of the metaphors and the visualisability of the connections that they create between mathematical objects and economic elements.

Conclusions

In this paper, I have argued that mathematics in economics involves the construction of metaphor and that mathematical metaphors have no strict equivalence in the field of literal expression. I have also introduced the concept of visualisability to economic science and linked it to the image-invoking property of metaphor. The construction of metaphors, both mathematical and literary, is indispensable to the development of knowledge in economics. Metaphors allow us to see things in new and different ways and provide an effective way of communicating the images that may come to the scientist in a moment of inspiration. Metaphor plays a valuable part in this process. This role is likely to be more effective and valuable if the metaphors of economics invoke clear, vivid images. It is important, therefore, for economists to consider the visualisability of the metaphors in economic theory. Vivid and visualisable mathematical economics may communicate new discoveries more effectively than metaphors in which the connections between mathematical objects and economic elements are faint and clouded by a gulf between principal and subsidiary subject. It must always be remembered, in addition to this, that complete discourse on the nature of economic reality—the entire theoretical structure of economic science—requires the deployment of a range of linguistic phenomena. The utilization of mathematics in economics and all of its strengths and weaknesses in this regard are language-based.

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[4] See Indurkhya's (1992, p. 316) comments on predictive analogy.

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