

DANGERS OF THE ONE-GOOD MODEL: BÖHM-BAWERK'S CRITIQUE OF THE "NAÏVE PRODUCTIVITY THEORY OF INTEREST"

BY
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I. INTRODUCTION

In the late nineteenth century, Eugen von Böhm-Bawerk's magisterial work (1881) on capital and interest provided the foundation upon which virtually all modern theories are built. In his first volume, *History and Critique of Interest Theories*, Böhm-Bawerk classified and (in his mind) refuted all previous explanations. Böhm-Bawerk thought a proper theory of interest must explain the apparent *undervaluation* of future goods. For example, if a machine is expected to yield annual rents of \$1,000 for ten years, why does it sell now for less than \$10,000? To answer this question was to provide a theory of interest, for only with such an undervaluation would it be possible for a capitalist to invest in machines, for example, and reap a flow of returns, over time, greater than his initial investment.

In particular, Böhm-Bawerk criticized what he called the "naïve productivity theory" of interest, which explained (and justified) the capitalists' income as a return to the "productivity of capital." Böhm-Bawerk famously argued that this theory was inadequate, since it only explained why capital goods possessed market value; by itself, the physical productivity of a capital good could not explain why its present purchase price was *lower* than the future revenues it was expected to yield. Referring to a tribal fisherman who initially lives hand-to-mouth before using a more capitalistic technique (an example familiar to economists of the time), Böhm-Bawerk argued:

Now let us turn to the second interpretation of which the naïve productivity theory is capable. Here the productive power ascribed to capital is, in the first instance, to be understood as physical productivity only, that is to say, a capacity on the part of capital to furnish assistance which results in the production of more goods or of

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better goods than could be obtained without its help. But it is assumed as self-evident that the increased product, besides replacing the costs of capital expended, must include a surplus of value. Just how convincing is this interpretation?

I grant without ado that capital actually possesses the physical productivity ascribed to it, that is to say, that more goods can actually be produced with its help than without. I will also grant . . . that the greater amount of goods produced with the help of capital has higher value than the smaller amount of goods produced without it. But there is not one single feature in the whole set of circumstances to indicate that this greater amount of goods must be worth more *than the capital consumed in its production*. And that is the feature of the phenomenon of excess value which has to be explained.

To put it in terms of Roscher's familiar illustration, I readily admit and understand that with the assistance of a boat and net one catches 30 fish a day, while without this capital one would have caught only 3. I readily admit and understand, furthermore, that the 30 fish are of higher value than the 3 were. But that the 30 fish must be worth more *than the pro rata portion of boat and net which is worn out in catching them* is an assumption which the conditions of the problem do not prepare us for, or even cause to appear tenable, to say nothing of making it obvious. If we did not know from experience that the value of the return to capital is regularly greater than the value of the substance of capital consumed, the naïve productivity theory would not furnish a single reason for regarding such a result as necessary. It might very well be quite otherwise. Why should not capital goods that yield a great return be highly valued on that very account and indeed, so highly that their capital value would be equal to the value of the abundance of goods which they yield? Why, for instance, should not a boat and net which, during the time that they last, help to procure an extra return of 2,700 fish be considered exactly equal in value to those 2,700 fish? But in that event, in spite of the physical productivity, there would be no excess value (I, pp. 93–94).

The modern reader may find Böhm-Bawerk's verbal reasoning—"literary economics"—difficult to evaluate. In particular, the modern economist may consider Böhm-Bawerk's critique of the "naïve productivity theory" to be rather anachronistic, in light of standard growth models that show a close relationship between the marginal productivity of capital and the equilibrium rate of interest.

This paper will attempt to illustrate Böhm-Bawerk's arguments through a simple, general equilibrium model in which the capital good is distinct from the consumption good. We will see that the standard one-good model of neoclassical growth theory obscures the subtleties in Böhm-Bawerk's critique. Only in a model with multiple goods can one fully appreciate the "Austrian" approach to capital and interest theory.

II. THE ONE-GOOD MODEL

The apparently straightforward connection between capital productivity and the interest rate is explained in David Romer's (1996) discussion of the Ramsey-Cass-Koopmans baseline growth model. In this model, a large number of identical firms have access to a production function $Y = F(K, AL)$, where K represents the firm's

capital stock, L represents the firm's labor supply, and A is the technology parameter measuring the "effectiveness of labor" (1996, p. 39). When describing firm behavior, Romer explains:

As described in Chapter 1, the marginal product of capital, $\partial F(K, AL)/\partial K$, is $f'(k)$, where $f(\cdot)$ is the intensive form of the production function. Because markets are competitive, capital earns its marginal product. And because there is no depreciation, the real rate of return on capital equals its earnings per unit time. Thus the real interest rate at time t is $r(t) = f'(k(t))$ (Romer 1996, p. 41).

Romer seems to claim that capitalists earn a return due to the technological productivity of their capital goods. Although he is not trying to "explain" interest in any philosophical sense, Romer's analysis seems quite compatible with the "naïve productivity theory" that Böhm-Bawerk criticized so harshly.¹ This raises the obvious question: Was Böhm-Bawerk's critique of the naïve productivity theory valid? After all, Romer has made no mistakes in deriving his equations.

The answer I will offer is that, paradoxically, both Böhm-Bawerk and Romer are correct. The fallacy against which Böhm-Bawerk warned is not relevant in an economy with a single good. The significance of Böhm-Bawerk's insights is only manifest in a model with distinct capital and consumption goods, as we will see in the next section.

III. DISTINCT CAPITAL AND CONSUMPTION GOODS

In this section we will examine a general equilibrium model with distinct capital and consumption goods. In contrast to Romer's typical setup, our approach will illuminate Böhm-Bawerk's critique of the naïve productivity theory of interest.

One Agent, Fixed Capital Stock

Imagine an economy consisting of one agent who lives for a finite number of time periods from 0 to T . The agent starts with an initial endowment, E , of the consumption good, and the agent possesses a fixed stock of machinery, \bar{K} , which does not depreciate with use. In each period the agent is endowed with a fixed labor supply, \bar{L} , which can be expended with no disutility.

¹ Romer also apparently embraces the "naïve productivity theory" in his empirical application of the Solow model. After obtaining an equation for the marginal product of capital, Romer says:

[This equation] implies that the elasticity of the marginal product of capital with respect to output is $-(1 - \alpha)/\alpha$. If $\alpha = 1/3$, a tenfold difference in output per worker arising from differences in capital per worker thus implies a hundredfold difference in the marginal product of capital. And since the return to capital is $f'(k) - \delta$, the difference in rates of return is even larger . . . [T]here is no evidence of such differences in rates of return. Direct measurement of returns on financial assets, for example, suggests only moderate variation over time and across countries (Romer 1996, p. 24).

This quote demonstrates that Romer believes the rates of return on financial assets, i.e., particular rates of interest, in some sense measure the physical productivity of capital goods. This seems to be the "naïve productivity theory" in modern form.

Each period the agent combines capital and labor to yield output² according to the function

$$Y_{t+1} = f(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}, \quad 0 \leq t \leq T-1,$$

where K_t and L_t are in the appropriate units (since capital, labor, and output are all distinct items). Suppose further that the agent has a utility function of the form

$$U_\tau = \sum_{t=\tau}^{t=T} \beta^{(t-\tau)} u(C_t), \quad 0 \leq \tau \leq T,$$

where $0 \leq \beta \leq 1$ is the discount on future utils, C_t is the amount of consumption in period t , and $u(\cdot)$ is differentiable, strictly increasing, and strictly concave.

Because of our grossly simplifying assumptions—namely, that labor carries no disutility, and that the stock of machines is fixed—it is obvious that the maximizing agent will produce $Y_t = f(\bar{K}, \bar{L}) = \bar{K}^\alpha \bar{L}^{1-\alpha} \equiv \bar{Y}$ units of the consumption good in every period t , where $1 \leq t \leq T$. For simplicity, we will assume that the initial endowment $E = \bar{Y}$. The agent's problem can thus be reduced to his formulation of a consumption plan when there is no production at all, and a fixed endowment of \bar{Y} units of the consumption good in each period t between 0 and T , inclusive. If we wish, we can let δ^C , where $0 \leq \delta^C \leq 1$, denote the depreciation rate of the stock of consumption goods. However, our choice of utility function ensures that, regardless of β and δ^C , the agent will choose $C_t = \bar{Y}$ for all t .³

The Fixed Capital Stock Model in a Market Setting

Because there are no externalities, we can easily transform the above model into one with separate capitalists and laborers who rent and sell machines and labor on a competitive market. If we normalize the spot price of consumption to 1 for all periods, then the equilibrium wage rate at time t is given by

$$w_t = \frac{f_2(K_t, L_t)}{1 + i_t} = \frac{(1 - \alpha)\bar{K}^\alpha \bar{L}^{-\alpha}}{1 + i_t}, \quad 0 \leq t \leq T-1,$$

where $f_2(\cdot, \cdot)$ denotes the partial derivative of f with respect to its second argument, and where i_t is the net real rate of interest. If we define p_y^x as the period x price of a unit of consumption good delivered in period y , then $i_t \equiv p_t^t/p_{t+1}^t - 1$. That is, the net rate of

² Notice that, in the spirit of "Austrian" capital theory, we have explicitly modeled the time involved in production; capital and labor applied in time t will only yield units of the consumption good in the following period $t+1$. This approach does not affect our main results.

³ Consider first the case where $\delta^C = 0$ and $\beta = 1$, so that the consumption good can be stored costlessly and future utils are not discounted. Even so, because of the strict concavity of $u(\cdot)$, we know that the maximizing agent will not choose to carry any units of consumption from some time t forward to any time $t + \tau$. A fortiori, the maximizing agent will never save any of the consumption good if storing it carries a penalty, i.e. when $\delta^C > 0$ and/or $\beta < 1$.

interest (at time t) is defined as the relative premium on consumption in period t versus consumption in period $t + 1$.

Similarly, we can define r_t as the rental price, in period t , for a unit of machinery for one period. That is, the owners receive r_t in period t for lending out a marginal unit of machinery and receiving it back, in perfect condition, one period later in $t + 1$. As with labor, the capitalists receive their discounted product in equilibrium, so that

$$r_t = \frac{f_1(K_t, L_t)}{1 + i_t} = \frac{\alpha \bar{K}^{\alpha-1} \bar{L}^{1-\alpha}}{1 + i_t}, \quad 0 \leq t \leq T - 1.$$

We now solve for the equilibrium rate of interest:

$$1 + i_t \equiv \frac{p_t^t}{p_{t+1}^t} = \frac{MU_t^t}{MU_{t+1}^t},$$

where MU_{t+1}^t is the marginal utility of consumption in period $t + 1$, as perceived in period t . With our choice of utility functions we know that $MU_{t+1}^t = \beta MU_{t+1}^{t+1}$. Finally, because $C_t = \bar{Y}$ for all t (as we argued in the single agent setting),⁴ we know that $MU_x^x = MU_y^y$ for any x, y . We therefore conclude that

$$1 + i_t = \frac{MU_t^t}{MU_{t+1}^t} = \frac{MU_t^t}{\beta MU_{t+1}^{t+1}} = \frac{1}{\beta}, \quad 0 \leq t \leq T - 1.$$

Discussion

Although the above model employed some very strong assumptions, it serves to conceptually distinguish the physical productivity of capital goods from the real rate of interest. Far from being equal to the “marginal productivity of capital,” the equilibrium interest rate is entirely a function of the representative agent’s subjective discount on future utils.⁵

To be sure, the (discounted) marginal productivity of machines determines the rental payments to the machine owners, just as the marginal productivity of labor determines the “rental price” of a worker’s body. But this does not in any way correspond to the rate of

⁴ To make the conceptual transition from the single agent setting to the current model as clear as possible, we can assume that the machine owners possess an initial endowment of consumption goods $\alpha \bar{K}^\alpha \bar{L}^{1-\alpha}$, while the laborers possess an initial endowment of $(1 - \alpha) \bar{K}^\alpha \bar{L}^{1-\alpha}$. In equilibrium, the machine owners will lend r_0 of their endowment in period 0 to entrepreneurs, laborers will lend w_0 , and they will consume the rest of their initial endowments. The entrepreneurs then use the loans to hire machines and labor, thus returning this real income right back to the original lenders in the form of rents and wages. Each period the entrepreneurs pay the interest on these original loans. In the final period, T , the entrepreneurs make their interest payment and return the principal as well. The point of this contrivance is to verify that $C_t = \bar{Y}$ for all t , just as in the single agent setting. (Recall that in period T , the world is ending and so $r_T = w_T = 0$.)

⁵ Proponents of the modern Austrian theory of interest, the “pure time preference theory,” should be careful in interpreting this result. There is a distinction between the rate of interest (i.e., the market’s discount on the price of future goods versus the same present goods) and the subjective time preference factor (i.e., the individual’s discount on future utility). The two are logically linked in this particular model only because of the strong assumption of a fixed capital stock (and hence constant level of consumption over time).

return on their *financial* capital. Consider the case where $\beta = 1$, so that there is no discount on future utils and the real interest rate is zero.⁶ In each period the machine owners receive $\bar{K}r_t = \alpha\bar{Y}$ in total payments; econometricians would find that the owners received an α share of total income.⁷ Nonetheless, the rate of return on financial assets is zero, despite the obvious physical productivity of the machines. What happens is that the total market value of the capital stock \bar{K} declines with each successive period, so that the rental payments accruing from ownership of the machinery exactly offset the decline in market value of the stock.⁸ This distinction—between physical productivity of capital goods and the rate of return on financial capital—is at the heart of Böhm-Bawerk's critique of the naïve productivity theory of interest. But the distinction cannot be seen in a single-good model.

Finally, we note that Böhm-Bawerk's own explanation of interest—namely, that it is intimately tied to the higher valuation of present over future goods—is valid in this model. Consider the more reasonable case where $\beta = 1/2$, so that the real interest rate is 100 percent. In equilibrium, it must be the case that capitalists who invest in machinery still receive the return from their machines' (discounted) marginal productivity, while also earning a net return of 100 percent per period on their financial assets.

We can most easily illustrate this by working backwards. First, define π_y^x as the period x price of a machine delivered at time y . At $t = T$, the stock of machinery will be useless; the world is ending and there is no point in producing for a non-existent future period $T + 1$. Consequently, $\pi_T^T = 0$. At time $T - 1$, a particular unit of machinery will still yield one last increment of marginal output, which the market will currently value at some constant $r_{T-1} \equiv \bar{r}$. Thus $\pi_{T-1}^{T-1} = \bar{r}$. In period $T-2$, a unit of machinery will yield an immediate return \bar{r} and will be worth π_{T-1}^{T-1} in the following period. Therefore, its present price π_{T-2}^{T-2} is $\bar{r} + \pi_{T-1}^{T-1}/(1 + i_{T-2}) = \bar{r} + \bar{r}/2 = (3/2)\bar{r}$. Thus we see that, when present consumption is twice as valuable on the margin as future consumption, the rate of return on financial assets is also 100 percent per period: A capitalist investing $3/2\bar{r}$ units of purchasing power in period $T-2$ would be able to purchase one unit of machinery. He could immediately lend it out to producers, earning an immediate return of \bar{r} (which could then be lent out on the loan market). In the following period, $T-1$, he would have $(1 + i_{T-2})\bar{r} + \pi_{T-1}^{T-1} = 2\bar{r} + \bar{r} = 3\bar{r}$ units of real purchasing power, i.e., a 100 percent return on his original investment.

⁶ To reiterate, Austrian readers should remember that the one does not always follow the other; it is perfectly possible to have a model in which there is no subjective discount on future utils, but nonetheless the market rate of interest is positive.

⁷ The reader should not worry that this analysis possibly conflates the return to the owners from their machines, and the interest return on their loan (made initially in period 0). In the text we are considering the case where $i_t = 0$, and so the machine owners' income is due (for $0 \leq t \leq T-1$) entirely to the rents earned by their machines. The owners have (financially) transferred, at a zero rate of interest, their initial endowment $\alpha\bar{K}^{\alpha-1}\bar{L}^{1-\alpha}$ to the last period T , when their machines will be worthless.

⁸ To be more specific: At $t = 0$, the stock of machinery will be valued at $\alpha\bar{Y}T$ units of present consumption. At $t = 1$, the entire capital stock will be valued at $\alpha\bar{Y}(T-1)$. So if capitalists invested $\alpha\bar{Y}T$ units of purchasing power at $t = 0$ in the stock \bar{K} of machines, they would receive a return of $\alpha\bar{Y}$ as rental payments in $t = 0$ (which could be invested at the prevailing interest rate of zero percent). Consequently, the total market value of their investment at $t = 1$ would be $\alpha\bar{Y}(1+0) + \alpha\bar{Y}(T-1) = \alpha\bar{Y}T$. In other words, the capitalists, although earning gross rents on their investment, would earn no net rents.

Variable Capital Stock?

Of course, the most obvious objection to the above analysis is that it seems to rule out, *a priori*, the possible influence of capital productivity on interest rates. When a modern economist states that the equilibrium interest rate equals the “marginal product of capital,” he or she is referring to the *net* technical productivity of capital. That is, the modern economist equates the interest rate with the percentage increase in physical output that will accrue from investing resources (such as labor) for an *additional* period.

This defense of current practice is certainly legitimate. Notice, however, that it is not relevant to Böhm-Bawerk’s critique. Böhm-Bawerk (in the passage quoted earlier) is not attacking productivity theories *per se*.⁹ Instead, he is attacking those “naïve” theories that explain interest as a return to the services of capital, in the same way that wages are a return to the services of labor. Such a naïve theory (which went hand in hand with the classical correspondence of land, labor, and capital with rent, wages, and interest) is based on a conceptual error: The increment in product (per unit time) yielded by an additional machine explains the rental (or hire) price of the machine, and *not* the percentage rate of return to someone who invests financial capital in that machine. If a fertile piece of land yields rents of \$10,000 per year, this fact alone tells us nothing about the *rate* of return to its owner; if the initial capitalized price of the land is \$100,000, then the rate of return is ten percent, while an initial price of \$200,000 will correspond to a return of only five percent.

I am not claiming that modern economists would dispute such elementary points.¹⁰ Rather, I claim that the typical one-good models of macroeconomics and growth theory obscure the issue and make even a truly naïve productivity theory seem correct. When the capital and consumption good are units of the same thing, there is no possibility of fluctuating exchange ratios allowing for different internal rates of return. If a tree yields 10,000 apples per year, this fact alone does not pin down the rate of return (in apples) to its owner, because the initial exchange value of the tree might be 100,000 or 200,000 apples. But if instead of a tree¹¹ we choose sheep as our example of a productive asset, and further suppose that 100 sheep will, if not consumed, produce 110 sheep in the following period, then the sheep-rate of interest is necessarily ten percent per period. But this isn’t because the naïve productivity theory is valid after all; rather, it is because the sheep-price of one sheep is always one (whereas the apple-price of a tree could in principle have any value whatever).¹²

⁹ This is obvious, for Böhm-Bawerk’s own explanation of interest famously involved the superior technical productivity of “roundabout” processes (II, pp. 273–83).

¹⁰ Indeed, one referee believes that even Böhm-Bawerk’s contemporaries committed no such logical fallacies. In any event, what Böhm-Bawerk meant by the naïve productivity theory certainly *is* fallacious, regardless of Böhm-Bawerk’s possible unfairness in attributing the view to particular authors.

¹¹ To avoid confusion, suppose that the tree produces *seedless* apples. There is thus no possibility of using the tree to produce more trees.

¹² In the appendix to Murphy (2003), I rigorously illustrate these arguments with a general equilibrium model in which labor and machines can be used to produce either a distinct consumption good or more machines.

IV. CONCLUSION

Böhm-Bawerk's critique of the naïve productivity theory of interest was valid, as the model in this paper has illustrated. The physical productivity of a capital good corresponds to its rental price, and not to the percentage rate of return to its owner. This elementary fact is obscured in modern one-good models, because they assume away the possibility of a capital good having a different exchange value from its product. This condition—which renders Böhm-Bawerk's critique irrelevant—is not due to any deep economic principle, but rather to the obvious fact that units of the exact same thing must exchange one-for-one on the market.

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