ON THE POSSIBILITY OF ASSIGNING PROBABILITIES TO SINGULAR CASES, OR: PROBABILITY IS SUBJECTIVE TOO!

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Introduction

IN THE SIXTH CHAPTER of the justly celebrated book *Human Action* (1996), Ludwig von Mises laid the theoretical groundwork for what has since evolved into the accepted Austrian theory of probability. Indeed, virtually every discussion of the theory of probability by Austrian economists since the publication of *Human Action* has unswervingly adhered to the theory of probability propounded by Ludwig von Mises in this chapter.[1]

For the development of his theory of probability, Ludwig von Mises drew heavily from the work of his brother, the positivist-mathematician Richard von Mises, who was one of the most outspoken proponents of what is known as the "relative frequency" or "frequentist" theory of probability. One of the most distinctive features of Richard von Mises's frequentist theory of probability, which was also subsequently adopted by Ludwig von Mises, is the idea that numerical probabilities can only be calculated for what Richard von Mises called "collectives" (and Ludwig von Mises called "classes") of repeatable events or phenomena (R. von Mises [1957] 1981, pp. 11–12; L. von Mises 1996, pp. 107–10).

This particular feature of both Richard and Ludwig von Mises's respective theories of probability completely rules out the possibility of assigning probabilities to individual events or phenomena which cannot

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[1]See, for example, Hoppe (1997 & 2007), Hülsmann (2003), Shostak (2002), and Rothbard (1956 & 1992). For a slightly different and somewhat ambivalent discussion of the theory and definition of probability, see Gordon (1991).

conceivably be assigned to a "collective" or a "class," and for which, consequently, relative frequencies cannot possibly be calculated. Both Ludwig and Richard von Mises were acutely aware that their respective frequentist theories of probability completely ruled out the possibility of calculating probabilities for singular, non-replicable cases. As Richard von Mises writes:

> When we speak of the "probability of death," the exact meaning of this expression can be defined in the following way only. We must not think of an individual, but of a certain class as a whole, e.g., "all insured men forty-one years old living in a given country and not engaged in certain dangerous occupations." A probability of death is attached to the class of men or to another class that can be defined in a similar way. We can say nothing about the probability of death of an individual even if we know his condition of life and health in detail. The phrase "probability of death," when it refers to a single person, has no meaning for us at all. (R. von Mises [1957] 1981, p. 11, emphasis added)

Following the lead of his brother, Ludwig von Mises also claimed that singular, non-replicable events and phenomena, (instances of what he called "case probability"), are "not open to any kind of numerical evaluation" (L. von Mises 1996, p. 113).[2]

It does not take any formal instruction in the philosophy of probability to recognize that the brothers von Mises are making a highly restrictive and exceptionally counterintuitive claim about the theoretical applicability of numerical probability. For, contrary to this claim of the brothers von Mises, most of us have encountered real world situations where numerical probabilities *are* assigned to future events which cannot conceivably be considered members of "collectives" or "classes." As a rather mundane example, consider the determination of numerical betting odds for boxing matches. It very often happens in the sport of boxing that two fighters meet in the ring having *never* fought one another in the past. Since the fighters never fought in the past, any bout between two such boxers cannot be considered a member of a "collective" or a "class" which would allow the

[2]The very same claim was later repeated by Rothbard almost verbatim:

> Richard von Mises has shown conclusively that numerical probability can be assigned only to situations where there is a class of entities, such that nothing is known about the members except they are members of this class, and where successive trials reveal an asymptotic tendency toward a stable proportion, or frequency of occurrence, of a certain event in that class. There can be no numerical probability applied to specific individual events.

(1956 p. 229, emphasis added). For similar statements, see also Rothbard (1992, p. 179), and Mises (1985, p. 308).

computation of *any* past relative frequencies of occurrence, let alone long-run relative frequencies of occurrence. As is well known, however, casinos and bookies do nevertheless assign numerical odds to these singular sporting events based upon *indirect* evidence (e.g., common opponents, injury reports, physical conditioning of the fighters, the fighters' ages and weights, perceived psychological advantages and disadvantages of each fighter, venue, etc.), and their odds are astoundingly accurate most of the time. Indeed, if casinos were not able to consistently generate very accurate numerical odds for these singular events, they would very quickly find themselves bankrupt and out of business.

According to the brothers von Mises, however, the assignment of a numerical probability to a singular case such as a boxing match is totally inappropriate and meaningless. What are we to make of this? Are the numerical odds assigned by casinos and bookies to singular boxing matches (and other singular events and phenomena, like the 2008 presidential election) absurd or meaningless, simply because they are not derived from long-run frequencies of "collectives" or "classes," as the von Mises brothers contend? If so, how can we explain the fact that casinos continuously generate such odds, and, more importantly, continue to make money year after year from wagers placed on the basis of those odds?

In this paper, I argue that Richard and Ludwig von Mises were mistaken to claim that numerical probabilities cannot be assigned to singular, non-replicable cases like boxing matches. I argue that Richard von Mises developed a demonstrably mistaken definition of probability, and that his mistaken definition inexorably led both von Mises brothers (and all Austrians who have followed their lead) to unduly proscribe the application of numerical probability to singular, non-replicable cases. Instead of defining probability in terms of purportedly "objective" long-run relative frequencies of "collectives" (as did Richard von Mises), I argue that consistent Austrians must define probability *subjectively;* that is, as a measure of *our* uncertainty about the likelihood of occurrence of some event or phenomenon, based upon evidence which need not derive solely from observations of past frequencies of "collectives" or "classes."

The paper is organized as follows. In the first section that follows I offer an historical and analytical sketch of Richard von Mises's frequentist definition of probability. In the second section that follows I attempt to reconstruct a proper and epistemologically defensible definition of probability that is radically different from that of Richard von Mises. I argue that the definition of probability necessarily depends upon whether the world in which we live is governed by time-invariant causal laws. In the third section that follows I argue that both the nature of human action and the relative frequency method for generating numerical probabilities demonstrate that the world is indeed governed by such time-invariant causal laws, and that this fact obliges us to adopt a subjective definition of probability. I conclude by arguing that if probability is defined subjectively, we need not condemn assigning numerical probabilities to singular, non-replicable cases.

Richard Von Mises's Frequentist Definition of Probability

Richard von Mises's developed his long-run frequentist definition of probability at a time when the definition and philosophical foundations for probability theory remained relatively undeveloped compared to most other branches of mathematics. As Harald Cramér describes the theoretical environment of the 1910's, the period in which Richard von Mises first developed his definition of probability:

> There was no commonly accepted definition of mathematical probability, and in so far as there were any definitions at all, they were clearly inadequate for the numerous applications that were made in fields such as population statistics, molecular physics, and many others. Moreover, with few exceptions, mainly belonging to the French and Russian schools, writers on probability did not seem to feel under any obligation to conform to the standards of rigor that were regarded as obvious in other parts of mathematics. (Cramér, 1953, p. 658)

For Richard von Mises, who would later write one of the most widely-read treatises on logical positivism (R. von Mises [1939] 1951), this lack of definitional consensus and standardized probabilistic methods signified that the discipline was not yet a true mathematical science.[3] As a doctrinaire

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[3]As Richard von Mises himself characterized the state of the discipline of probability in 1919:

> He who follows the development of the calculus of probability in the last decades cannot deny that this branch of the science of mathematics is behind all others in two respects. The analytical theorems of the calculus of probability are lacking—except for few works by Russian mathematicians—the precision of formulation and of proof-method which has been a matter of course for long in other parts of analysis. And there is, in spite of some valuable beginnings, almost no clarity about the foundations of the calculus of probability as a mathematical discipline; this is all the more surprising as we are living not only in an age of vivid interest in questions of axiomatics in mathematics but also in a period of increasing use of the calculus of probability in various fields of application. [Richard von Mises, 1919a, pp. 1-2. Quoted and translated by Siegmund-Schultze (2006, p. 438)]

On this, see also the preface to the third German edition of *Probability, Statistics and Truth* ([1957] 1981).

logical positivist, Richard von Mises sought to correct what he perceived to be scientific deficiencies of probability theory by supplying what he called a "scientific definition of the *concept* of probability," (R. von Mises [1957] 1981, p. 1. emphasis in original), that would help to transform probability theory into "a part of descriptive physical science" (Weatherford 1982, p. 145). Thus, Richard von Mises's goal was to transform probability theory into another branch of "objective" empirical science, based upon the positivist assumption that the "same fundamental methods are applicable in the theory of probability as in all other sciences" (R. von Mises [1957] 1981, p. 31).[4]

The definition of probability subsequently advanced by Richard von Mises was crucially affected by this attempt to transform probability into a positive "science" like every other positive natural science. In the first place, this led him to assume that the theory of probability, like the natural sciences, must be based upon actual *a posteriori* observations, and must "draw conclusions which can be tested by comparison with experimental results" (R. von Mises [1957] 1981, p. 31). This desire to construct a theory of probability which shared the positivist method, in turn, quite naturally predisposed him to the relative frequency method for determining probabilities that was "given explicit articulation by John Venn" in 1866 (Kyburg Jr., 1990, p. 78). To a man with as dominating positivist inclinations as Richard von Mises, the relative frequency method for determining probabilities had a crucially appealing characteristic: it afforded a *replicable* and *verifiable* method for experimentally determining probabilities. It was also a method with which Richard von Mises was intimately acquainted, (having made important contributions to the method in another famous 1919 article), a method that offered a sophisticated mathematical alternative to the classical combinatorial method for determining probabilities, and a method that dovetailed perfectly with the then-rapidly expanding field of statistics.

Not surprisingly, the confluence of these various factors led Richard von Mises to advance a definition of probability that was built upon the *method* for determining probabilities that was most amenable to the positivists of Richard von Mises's generation; namely, Venn's method for calculating a relative frequency of occurrence for a *finite* series of actual observations. Richard von Mises's novel contribution, however, was to transform this relative frequency *method* for determining probabilities into a *conceptual definition* of probability. In other words, Richard von Mises transformed the relative frequency method for determining probabilities as it necessarily was and still is used in the real world, (i.e., to calculate a ratio of occurrence using

[4]Richard von Mises's decision to utilize the motto "The true method of philosophy is none other than that of the natural sciences" for his book Positivism is instructive here. For this quote, see Barry Smith (1994, p. 31).

a *finite* series of *actual* observations), into a *conceptual* definition of probability grounded on the purely hypothetical idea of extending a sequence of observations *indefinitely* or *infinitely*.[5] The result, as Richard von Mises himself summarizes, was the famous long-run relative frequency definition of probability:

> "The probability of an attribute…within a collective is the limiting value of the relative frequency with which this attribute recurs in the indefinitely prolonged sequence of observations." (R. von Mises, [1957] 1981, p. 221).[6]

It is important to note at this point that Richard von Mises's definition rules out the application of the probability calculus to singular events and phenomena *simply by definition*. For, if probability *itself* is defined as a ratio of occurrence in an indefinitely or infinitely prolonged sequence of observations within a "collective," this definition necessarily excludes from the entire concept of probability all events and phenomena which are not, and never could be, members of any sequence or "collective" at all—like singular boxing matches. This is precisely the conclusion reached by Richard von Mises:

> We state here explicitly: The rational concept of probability, which is the only basis of probability calculus, applies only to problems in which either the same event repeats itself again and again, or a great number of uniform elements are involved at the same time. Using the language of physics, we may say that in order to apply the theory of probability we must have a practically unlimited sequence of uniform observations. (R. von Mises [1957] 1981, p. 11)

Thus, Richard von Mises's famous claim that numerical probability cannot be applied to singular events and phenomena ultimately stems from his positivism-inspired relative frequency definition of probability. In the next section I attempt to reconstruct an epistemologically defensible definition of probability from the ground up that, as will be seen later, undercuts Richard von Mises's definition.

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[5]It is important to take note here of the fact that it is literally impossible in practice to extend a series of observations indefinitely or infinitely. Man is obviously not in a position to be able to gather sequences of observations indefinitely or infinitely; on the contrary, man necessarily has access only to finite series of observations that he makes in the real world. Thus, paradoxically, while Richard von Mises's goal was to create a "scientific" discipline of probability based upon actual observations, he ended up creating a conceptual definition of probability based upon an action that is literally impossible for human beings to perform.

[6]For the full explanation and development of this definition, consult the first lecture in (R. von Mises, [1957] 1981) entitled "The Definition of Probability."

Causal Determinism and the Definition of Probability

With the foregoing historical and analytical sketch of Richard von Mises's relative frequency definition of probability in mind, I will now proceed to reconstruct a proper definition of probability. In order to accomplish this task, however, it is first necessary to make a preliminary investigation into the nature of the subject matter for which we are seeking a proper definition. For, in the absence of even a general and common sense idea of what probability is, we will lack any criteria by which to judge various definitions of probability—including Richard von Mises's definition. I will begin this section, therefore, with some preliminary observations about the nature of probability that will play a critical role in my later criticism of Richard von Mises's definition of probability.

Uncertainty

It is vital to note that when we deal with the subject of probability we must necessarily and concomitantly deal with the subject of *uncertainty*. The term "probable" applies to statements and facts about which we are uncertain—the word does not apply to statements and facts about which we are already absolutely certain. As Ludwig von Mises eloquently explains:

> A statement is probable if our knowledge concerning its content is deficient. We do not know everything which would be required for a definite decision between true and not true. But, on the other hand, we do know something about it; we are in a position to say more than simply *non liquet* or *ignoramus*. (L. von Mises 1996, p. 107)

It is precisely because man is uncertain about many past and future events, outcomes, statements and phenomena that the need for probability arises. Indeed, if the world were such that man already possessed certain knowledge about all past and future events and phenomena, he would have no need to resort to round-about methods for measuring the likelihood of those events and phenomena, for he would already possess *certain* knowledge about both everything that happened in the past and everything that will happen in the future. Hence, there would be no such thing as probability or *inferential* statistics in a world in which man was omniscient; on the contrary, in such a world there would only exist an infinite collection of facts, propositions and (non-inferential) statistics about which man was already absolutely certain.

Because man obviously is *not* omniscient, however, he often has a need or desire to develop round-about methods which are capable of measuring his uncertainty about past and future events and phenomena. This is precisely the function that probability, (among other methods),[7] plays for man in his quest to deal with uncertainty. As Anthony O'Hagan succinctly explains:

> Probability is a measure of uncertainty, and the theory of probability, with the related subject of statistics, provides a scientific methodology for evaluating probabilities. Its importance can scarcely be overstated. The abundance of things about which we are uncertain, and the way in which our future actions, prosperity and even existence depend on uncertainties, is reinforced by every news report. Accurate weighing of uncertainties is vital to good decision-making at every level—personal, national, or international. (O'Hagan 1988, p. 2)

Where Does Uncertainty Come From?

Because the very existence of probability and its employment by man is predicated on the existence of uncertainty, it is thus necessary for us to find out why, specifically, man is uncertain about future and past events and phenomena. Why, for example, does man construct and employ probabilistic methods as a round-about measure of the likelihood that a flipped coin will come up heads? Does man utilize these round-about methods only because he is mentally incapable of comprehending the myriad causal factors involved in the process of flipping a coin? Or, does man utilize these round-about methods because the system, or the parts of the system, he seeks to understand is *itself* indeterministic, uncaused, and thus inherently unpredictable?

The answer to this question is of unparalleled philosophical importance for the present investigation. For, as I hope to show here following the lead of I.J. Good, if uncertainty is ascertained to derive solely from our limited mental capacity to comprehend all of the relevant factors involved in any given process, *while the process itself is governed by causally deterministic laws*, then this will force us to adopt a subjective definition of probability. If, on the other hand, uncertainty is ascertained to derive from the natural world itself, (that is, the processes of the natural world are subject to random, uncaused

[7]While man can and does use probability as a means to overcome uncertainty, this by no means implies that probability is the only means by which man can overcome uncertainty, however. Man is capable of acquiring knowledge about himself as an acting agent, for example, that is known to be absolutely and irrefutably true simply through the use of his reason, and without the aid of probability. On this, see in particular Hoppe (1997). Those probabilists, (and statisticians in particular), therefore, who would concur with Winkler's claim that "We must assign a central role to probability for all knowledge and understanding" (1990, p. 128) would do well to ask themselves whether Winkler's purportedly true claim is itself derived from probability.

outcomes), then we are free (but are not required) to adopt a non-subjectivist definition.

The vital issues that are involved here can perhaps best be illuminated by means of a practical example. Suppose that man has a desire to predict the weather in a particular part of the world. Quite obviously, this desire to predict the weather in the future ultimately would derive from the fact that man is *uncertain* about future weather patterns. What is not immediately obvious, however, is *why* man is uncertain about the weather in the future. Does his uncertainty derive simply from the fact that future weather patterns involve an almost infinitely complex array of various causal factors, and man's limited mental and technological capacity is not such that would allow him to know all of those various causal factors in advance? Or, does man's uncertainty about future weather patterns derive from a *fundamental indeterminism*, (i.e., uncaused randomness) in the weather system *itself*? In short, does uncertainty derive from A) man's limited knowledge in a causally deterministic world, as Ludwig von Mises held, or, B) does the world *itself* contain an element of *uncaused* randomness, as Richard von Mises held?[8]

The relevance of this question to the present inquiry arises from the fact that, and this is vital, in a causally deterministic world (that is, *a world in which every event and phenomenon in the world has an antecedent and time-invariant cause, or causes*), uncertainty and randomness *cannot* be "properties" of the physical world itself.[9] On the contrary, in a causally deterministic world, every event and phenomenon that occurs in the world has, *by definition*, a prior and *certain* cause, or causes. Hence, in a causally deterministic world, all uncertainty about events and phenomena in the world must derive *solely from human ignorance* of the deterministic causes of those events and phenomena. As I.J. Good explains:

> [I]f we assume determinism we can get physical probabilities only by having an incompletely specified physical setup. In this incomplete

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[8]On Richard von Mises's espousal of indeterminism, see lecture 6 in R. von Mises (1981 [1957])

[9]It is important to note here that the idea of "causal determinism," (i.e., the idea that every event and phenomenon that occurs in the world has an antecedent and sufficient cause), does not imply that man has no free will. For, while causal determinism does hold that there are causes for all human actions, (namely, the ideas and intentions of the actors involved), this by no means implies that man is not free in the metaphysical sense. For the distinction between those often referred to as "hard determinists" (i.e., those who deny free will) and "causal determinists" (i.e., those who hold that everything that happens has a cause), see the useful and lucid discussion in Lacy (1976) pp. 114-118. The present inquiry, however, is only concerned with the question "Does everything that occurs in the world have a cause?," and is not concerned to answer the metaphysical question "Are man's actions predetermined?"

> specification there must be probabilities. If we are determinists we must attribute these latter probabilities to our own ignorance and not merely to something basic in nature "out there." Whether or not we assume determinism, every physical probability *can* be interpreted as a subjective probability or as a credibility. *If we do assume determinism, then such an interpretation is forced upon us.* (Good, 1959, p. 447. emphasis added)

Hence, as Good explains here, if the world is governed by deterministic laws, we are forced to say that any uncertainty man might have about events and phenomena in that world would derive from man's limited knowledge, rather than from the world itself. *Thus, probability in such a world would necessarily be a subjective numerical measure of man's beliefs about the world, rather than an "objective" measure of a property that exists in the world, because all outcomes, events and phenomena in a causally deterministic world have absolutely certain causes.* If man were in a position to know in advance all of the time-invariant causal factors affecting any given event or phenomenon, he would not have to resort to the round-about methods of probability to predict outcomes. He would know in advance, and for certain, what was going to occur. As Laplace famously observed:

> Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes. (Laplace [1814] 1956, p. 1325)

Therefore, if all events and phenomena that occur in the world are indeed causally determined, then we have to abandon Richard von Mises's relative frequency definition of probability in favor of a subjective definition. For, if the events and phenomena that occur in the world are all *caused*, then we are forced to ascribe any uncertainty man might have about those causes or outcomes to his own ignorance, and this necessarily implies that probability, as a numerical measure of uncertainty, is necessarily a *subjective numerical statement of man's beliefs about the operant causes in the world*. We would be forced to re-label Richard von Mises's purported "conceptual *definition*" as a "conceptual *method*" for generating probabilities, rather than the very definition of probability itself.[10]

[10]We use the label "conceptual method" rather than simply "method" here, because, as was noted above (see note 5), it is literally impossible for man to collect indefinite or infinite series of observations in the real world. As such, it is more appropriate to treat the long-run relative frequency method described by Richard von Mises as a "conceptual method" rather than an actual method man can employ. Since man cannot actually use

In the next section, I claim that the events and phenomena that occur in the world are indeed governed by causally deterministic laws, which necessarily implies that we must adopt a subjective definition of probability.

The World Is Governed By Time-Invariant Causal Laws

As was just seen, the definition of probability hinges on whether the events and phenomena that occur in the world are governed by causal laws, or whether there exists in the world an element of uncaused randomness. If we conclude that the events and phenomena that occur in the world are indeed governed by causal laws, then we will be forced to ascribe any uncertainty man has about those events and phenomena to man's ignorance, and not to some "objective" characteristic of the world. Probability would then necessarily be a *subjective numerical measure of man's uncertainty* about the causal forces operating in the world. In this section I aim to demonstrate that the events and phenomena that occur in this world are indeed governed by time-invariant causal laws.

Human Action Presupposes a Causally Deterministic World

The task of establishing that the world in which human beings live and act is governed by time-invariant causal laws has been greatly aided by the work of Ludwig von Mises. This should not be particularly surprising, since Ludwig von Mises was, in fact, a determinist (Gordon 1996, p. 53). Ludwig von Mises's position with regard to causation was that everything that occurs in the world necessarily has an antecedent cause:

> No change occurs that would not be the necessary consequence of the preceding state. All facts are dependent upon and conditioned by their causes. No deviation from the necessary course of affairs is possible. Eternal law regulates everything. (L. von Mises 1985, p. 74).

According to Ludwig von Mises, moreover, man is in a position to know *with certainty* that the world in which he lives and acts is in fact governed by causal laws. Man knows this because human action is only possible in a world so constituted. As Ludwig von Mises writes:

> Acting requires and presupposes the category of causality. Only a man who sees the world in the light of causality is fitted to act. In this sense we may say that causality is a category of action. The category *means and end* presupposes the category *cause and effect*. In a world without causality and regularity of phenomena there would be no field for human

this as a method in his scientific pursuits, it is best categorized as a "conceptual method" only. I am grateful to Robert A. Crovelli for this point.

> reasoning and human action. Such a world would be a chaos in which man would be at a loss to find any orientation and guidance. Man is not even capable of imagining the conditions of such a chaotic universe. (L. von Mises 1996, p. 22. Emphasis in original)

The idea that causality is a category of human action has been extensively developed and analyzed by Hans-Hermann Hoppe. As Hoppe cogently explains, human action presupposes the existence of "constant, time-invariantly operating causes," and this fact establishes, *a priori*, that the world is indeed governed by such causes:

> [T]he principle of causality must be understood as implied in our understanding of action as an interference with the observational world, made with the intent of diverting the "natural" course of events in order to produce a different, preferred state of affairs, i.e., of making things happen that otherwise would not happen, and thus presupposes the notion of events which are related to each other through time-invariantly operating causes. An actor might err with respect to his particular assumptions about which earlier interference produced which later result. But successful or not, any action, changed or unchanged in the light of its previous success or failure, presupposes that there are constantly connected events *as such*, even if no particular cause for any particular event can ever be preknown to any actor...It is simply by virtue of acting and distinguishing between successes and failures that the a priori validity of the principle of causality is established; even if one tried, one could not successfully refute its validity. (Hoppe 1995, pp. 77–78. Emphasis in original)[11]

Furthermore, the principle of causality must be regarded as applying not just to the realm of human action, but rather to the non-human world as well. As Brand Blanshard notes in this regard, "If the mastery of causal laws gives control and understanding, *it is because such laws are more than devices of our own*. They belong to nature; they are part of the network, the set of objective ties, that bind events together" (Blanshard 1964, p. 445. Emphasis added).

These observations have a vital bearing on the definition of probability. For, as was indicated above, the definition of probability hinges upon whether the world is governed by deterministic causal laws. And, as Ludwig von Mises and Hans-Hermann Hoppe have demonstrated, since human action necessarily presupposes the existence of just such deterministic and time-invariant causal laws, we must therefore conclude that probability is a *subjective* numerical measure of our own ignorance about those constant and time-invariant causes, rather than a feature of the world itself.

[11] In this connection, see also Hoppe's brilliant article "Is Research Based on Causal Scientific Principles Possible in the Social Sciences?" (2006).

Richard Von Mises's Relative Frequency Method Presupposes a Causally Deterministic World

Further proof of the fact that the world is governed by time-invariant causal laws can be obtained by taking a closer look at the relative frequency method for generating numerical probabilities. It is my contention here that it would be absurd and self-contradictory to attempt to employ the relative frequency method for generating numerical probabilities unless the phenomenon under study is assumed to be governed by time-invariant causal laws.

I shall begin my argument by making some preliminary observations about the relative frequency method for generating numerical probabilities. As indicated above, the relative frequency method for generating numerical probabilities (which was transformed by Richard von Mises into the definition of probability) begins with a series of events that are virtually identical to one another. As Richard von Mises states, "in order to apply the theory of probability we must have a practically unlimited sequence of uniform observations" (R. von Mises [1957] 1981, p. 11). This criterion for the employment of the relative frequency method has two important implications: 1) the events must be *virtually* identical with one another, such that they can conceptually be categorized together as members of a "collective," and 2) the events must not be *exactly* identical to one another in every conceivable way, or else there would be no variation in outcome.

With respect to the latter of these implications, there must be some variation in the events one is observing, or else one will always end up with exactly the same outcome. For example, if one were hypothetically able to flip a coin in *exactly* the same way (with the *exact* same forces involved, the *exact* same torsion, the *exact* same coin, *et cetera ad infinitum*) one would end up with the exact same outcome every single time. There would be nothing *probable* at all about the outcomes of *absolutely identical* events; on the contrary, there would be *certainty* that one would obtain the same result every time. In practice, moreover, it should be clear that when one deals with observations in the real world one deals with events that are different from one another in at least some respects. At the very minimum, each event in the world occurs at a different time and at a different point in space. One never deals with events in the world that are *exactly* the same in every conceivable way.

With respect to the former of these implications, on the other hand, the relative frequency method requires that the events be similar to one another in *virtually* every way, or else the events cannot be classified together as members of the same "collective." As Richard von Mises uses the term "collective," he means "*a sequence of uniform events* or processes which differ by certain observable attributes, say colors, numbers, or anything else" (R. von Mises [1957] 1981, p. 12. Emphasis added). Events that are not sufficiently similar to one another, (like the tossing of a fair coin and the tossing of a weighted coin), cannot be classified together as members of the same collective.

This latter implication is of critical importance for establishing that the world is governed by time-invariant causal laws. For, in order to classify several discreet events together as a "collective," *one must assume that the underlying causal factors affecting each of the events are virtually identical in every way.* Were one not to make the assumption that the same causal factors were operating on each of the events in virtually exactly the same way, one would not be in a position to say that the events were sufficiently similar to one another to be classified as members of the same "collective." One would not be in a position to decide whether the events were members of the same "collective," or whether they were so different from one another as to be logically incommensurable.[12]

An example will help to illuminate this point. Imagine that one were to observe the tossing of a die (singular for dice) over time. In order to be able to conceptually classify these discreet and sequential tosses of the die into a "collective" and calculate a meaningful relative frequency of occurrence for them, one must assume that the same time-invariant casual laws are affecting each toss of the die in the same way. Otherwise, if one were not to make this assumption, one would not be in a position to know whether they were instances of like phenomena, or whether they were logically incommensurable. If gravity and torsion were assumed to affect each toss of the die in radically different ways each time the die was tossed, for example, each toss would be logically incommensurable from the others, and any relative frequency of occurrence that one might calculate for the events would thus be meaningless. This is especially clear if past relative frequencies are used to predict future events and phenomena, because unless one assumes that future events and phenomena will be affected by causal forces in the same way that past events and phenomena have been affected, relative frequencies of occurrence for past observations would have absolutely no meaning for future observations.

[12]There is thus an inherent degree of subjectivity in the determination of whether two or more events in the world are sufficiently similar to one another to be classified as members of a "collective." Because events are necessarily different from one another in at least some respects, man must make a subjective judgment about whether these differences are so great as to make the events conceptually incommensurable, or whether they are so minor as to be able to treat the events (at least conceptually as) as identical.

Hence, Richard von Mises's own favored method for generating numerical probabilities about collectives must presuppose that the world in which one employs the method is governed by time-invariant causal laws. And, as was seen in the previous section, this fact establishes that the definition of probability must be subjective.

Conclusion

We are now in a position to return to the question posed in the introduction; namely, is it meaningless or absurd to calculate numerical probabilities for singular events and phenomena like boxing matches, as the brothers von Mises contend? Heretofore, I have not specifically broached the question. Instead, I have argued that the definition of probability depends upon whether the world is governed by time-invariant causal laws, or whether there exists in the world an element of uncaused randomness. I have argued that if the world is governed by time-invariant causal laws, then we must ascribe any uncertainty man might have about those causes to human ignorance alone and not to some property of the world, and that we would be consequently obliged to define probability as a subjective measure of man's beliefs about the causal factors at work in the world. Furthermore, I have argued that both human action and the relative frequency method for generating numerical probabilities presuppose that the world is indeed governed by time-invariant causal laws.

With these observations in mind, I am now in a position to explicitly address the question of whether calculating numerical probabilities for singular events and phenomena are absurd and meaningless. The answer is that it is neither absurd nor meaningless to calculate numerical probabilities for such events. Since probability necessarily deals with events and phenomena about which man is uncertain, it would be dogmatic and unjustifiable to claim, as have the von Mises brothers, that the only method that can ever be legitimately employed by man to quantify his uncertainty about the causal factors at work in the world is the relative frequency method.

As was seen, Richard von Mises's condemnation of non-frequentist methods for calculating numerical probabilities was based solely upon his exceptionally restrictive definition of probability—a definition that is demonstrably untenable in a world that is governed by time-invariant causal laws. If we reject his definition, and substitute a subjective definition in its place, we concomitantly undercut his sole criticism of non-frequentist methods and open up the possibility of employing other methods for numerically measuring our uncertainty about the causal factors operating in the world. This would include, for example, methods for generating numerical probabilities for singular boxing matches, singular football matches, singular international wars, singular elections, *et cetera*. This conclusion, moreover, comports with the common sense observation that the numerical odds generated for singular boxing matches by Las Vegas bookies are not merely meaningless and absurd number games that are completely unrelated to the real world. They are, on the contrary, numerical measurements about the likelihood of occurrence that, while generated by weighing evidence that is unrelated to past relative frequencies, are remarkably accurate much of the time. And this is true, in the final analysis, because probability, like economic value, is a subjective measurement of man's beliefs about the world in which he lives.

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